Week 11: *WTP & Simulations*

EMSE 6035: Marketing Analytics for Design Decisions

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Week 11: *WTP & Simulations*

1. Computing "Willingness to Pay" (WTP)

2. Incorporating Uncertainty via Simulation

3. Directly Estimating WTP

BREAK

4. Simulating Market Shares

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Willingness to Pay (WTP)

$$\tilde{u}_j = \alpha p_j + \boldsymbol{\beta} x_j + \tilde{\varepsilon}_j$$

$$\boldsymbol{\omega} = \frac{\boldsymbol{\beta}}{-\alpha}$$

Notational convention: hats mean "estimated"

Model

$$\tilde{u}_j = \alpha p_j + \boldsymbol{\beta} x_j + \tilde{\varepsilon}_j$$

$$\boldsymbol{\omega} = \frac{\boldsymbol{\beta}}{-\alpha}$$

Estimated Model

$$\tilde{u}_j = \hat{\alpha} p_j + \hat{\boldsymbol{\beta}} x_j + \tilde{\varepsilon}_j$$

$$\hat{\boldsymbol{\omega}} = \frac{\hat{\boldsymbol{\beta}}}{-\hat{\alpha}}$$

Practice Question 1

Suppose we estimate the following utility model describing preferences for cars:

$$\tilde{u}_j = \alpha p_j + \beta_1 x_j^{\text{mpg}} + \beta_2 x_j^{\text{elec}} + \varepsilon_j$$

The estimated model produces the following coefficients:

- α: -0.7
- β_1: 0.1
- β_2: -4.0

a) Use the estimated coefficients to compute the WTP for fuel economy and electric car vehicle type.

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Simulating uncertainty

We can use the coefficients and hessian from a model to obtain draws that reflect parameter uncertainty

```r
beta <- c(-0.7, 0.1, -4.0)

hessian <- matrix(c(
    -6000,   50,   60,
       50, -700,   50,
       60,   50, -300),
    ncol = 3, byrow = TRUE)
```

```r
covariance <- -1*solve(hessian)
draws <- MASS::mvrnorm(10^4, beta,
covariance)

head(draws)
```

```
#>            [,1]       [,2]      [,3]
#> [1,] -0.6892878 0.12494924 -4.009292
#> [2,] -0.7153489 0.10572215 -3.971951
#> [3,] -0.7009624 0.11457475 -3.980457
#> [4,] -0.6949320 0.09787724 -3.979199
#> [5,] -0.6871134 0.04347226 -4.030089
#> [6,] -0.6990488 0.05952774 -4.047897
```

Computing WTP with draws

$$\hat{\omega} = \frac{\hat{\beta}}{-\hat{\alpha}}$$

```
draws_other <- draws[,2:ncol(draws)]
draws_price <- draws[,1]
draws_wtp <- draws_other / (-1*draws_price)
head(draws_wtp)
```

```
#>            [,1]      [,2]
#> [1,] 0.18127295 -5.816572
#> [2,] 0.14779103 -5.552467
#> [3,] 0.16345350 -5.678560
#> [4,] 0.14084434 -5.726026
#> [5,] 0.06326795 -5.865246
#> [6,] 0.08515534 -5.790578
```

Mean WTP with confidence interal

```
logitr::ci(draws_wtp)
```

```
#>         mean       lower      upper
#> 1  0.1423682  0.03380317  0.2499869
#> 2 -5.7154451 -5.97876744 -5.4608139
```

Practice Question 2

Suppose we estimate the following utility model describing preferences for cars:

$$\tilde{u}_j = \alpha p_j + \beta_1 x_j^{\text{mpg}} + \beta_2 x_j^{\text{elec}} + \varepsilon_j$$

The estimated model produces the following coefficients and hessian:

- α: -0.7
- β_1: 0.1
- β_2: -4.0

$$\nabla_\beta^2 \ln(\mathcal{L}) = \begin{bmatrix} -6000 & 50 & 60 \\ 50 & -700 & 50 \\ 60 & 50 & -300 \end{bmatrix}$$

a) Generate 10,000 draws of the model coefficients using the estimated coefficients and hessian. Use the `mvrnorm()` function from the `MASS` library.

b) Use the draws to compute the mean WTP and 95% confidence intervals of WTP for fuel economy and electric car vehicle type.

Computing WTP from an estimated model

1. Open `logitr-cars`
2. Open `code/6.1-compute-wtp.R`

Your Turn

10:00

As a team, compute the WTP from an estimated model you used in your pilot analysis report

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Willingness to Pay (WTP)

"Preference Space"

$$\tilde{u}_j = \alpha p_j + \boldsymbol{\beta} x_j + \tilde{\varepsilon}_j$$

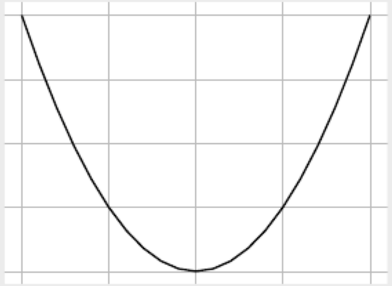
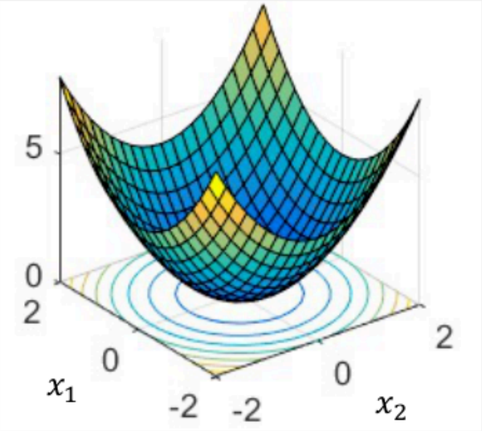
"WTP Space"

$$\boldsymbol{\omega} = \frac{\boldsymbol{\beta}}{-\alpha}$$

$$\boldsymbol{\lambda} = -\alpha$$

$$\tilde{u}_j = \boldsymbol{\lambda}(\boldsymbol{\omega} x_j - p_j) + \tilde{\varepsilon}_j$$

WTP space models have **non-convex** log-likelihood functions!

Number of dimensions	First order condition	Second order condition	Example
One	$\frac{df(x^*)}{dx} = 0$	$\frac{d^2f(x^*)}{dx^2} > 0$	
Multiple	"Gradient" $\nabla f(x_1, x_2, \dots x_n)$ $= \left[\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \dots, \frac{\partial f}{\partial x_n}\right]$ $= [0,0, \dots, 0]$	"Hessian" $\nabla^2 f(x_1, x_2, \dots x_n)$ $= \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \cdots & \frac{\partial^2 f}{\partial x_n \partial x_1} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 f}{\partial x_1 \partial x_n} & \cdots & \frac{\partial^2 f}{\partial x_n^2} \end{bmatrix}$ Must be "positive definite"	

WTP space models have non-convex log-likelihood functions!

Use multi-start loop with random starting points

Computing WTP from an estimated model

1. Open `logitr-cars`
2. Open `code/6.2-model-wtp.R`

Your Turn

10:00

As a team, re-estimate the main model you used in your pilot analysis report, but now in the WTP space.

Try plotting your results (see `6.3-plot-wtp.R` for examples)

Break

05:00

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We want to find β by maximizing the log-likelihood

$$\tilde{u}_j = \boldsymbol{\beta}'\mathbf{x}_j + \tilde{\varepsilon}_j$$

$$= \boxed{\beta_1} x_{j1} + \boxed{\beta_2} x_{j2} + \ldots + \tilde{\varepsilon}_j$$

Weights that denote the *relative* value of attributes $x_{j1}, x_{j2}, \ldots$

Estimate β_1 , β_2 , ... , by minimizing the negative log-likelihood function:

$$\text{minimize} - \ln(\mathcal{L}) = -\sum_{j=1}^{J} y_j \ln\left[P_j(\boldsymbol{\beta}|\mathbf{x})\right]$$

with respect to $\boldsymbol{\beta}$

$y_j = 1$ if alternative j was chosen
$y_j = 0$ if alternative j was not chosen

$$P_j = \frac{e^{v_j}}{\sum_{k=1}^{J} e^{v_k}} = \frac{e^{\boldsymbol{\beta}'\mathbf{x}_j}}{\sum_{k=1}^{J} e^{\boldsymbol{\beta}'\mathbf{x}_k}}$$

Simulate Market Shares

1. Define a market, X
2. Compute shares:

$$\hat{P}_j = \frac{e^{\hat{\boldsymbol{\beta}}' \boldsymbol{X}_j}}{\sum_{k=1}^{J} e^{\hat{\boldsymbol{\beta}}' \boldsymbol{X}_k}}$$

Simulate Market Shares

$$
\begin{aligned}
\hat{v} &= \hat{\boldsymbol{\beta}}'\mathbf{x} \\
&= \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \vdots & \vdots & \dots & \vdots \\ x_{J1} & x_{J2} & \dots & x_{Jn} \end{bmatrix} \begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \\ \vdots \\ \hat{\beta}_n \end{bmatrix} \\
&= \begin{bmatrix} \hat{\beta}_1 x_{11} + \hat{\beta}_2 x_{12} + \dots + \hat{\beta}_n x_{1n} \\ \hat{\beta}_1 x_{21} + \hat{\beta}_2 x_{22} + \dots + \hat{\beta}_n x_{2n} \\ \vdots \\ \hat{\beta}_1 x_{J1} + \hat{\beta}_2 x_{J2} + \dots + \hat{\beta}_n x_{Jn} \end{bmatrix} = \begin{bmatrix} \hat{v}_1 \\ \hat{v}_2 \\ \vdots \\ \hat{v}_J \end{bmatrix}
\end{aligned}
$$

Simulate Market Shares

$$\hat{v} = \widehat{\boldsymbol{\beta}}'\mathbf{x}$$

$$= \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \vdots & \vdots & \dots & \vdots \\ x_{J1} & x_{J2} & \dots & x_{Jn} \end{bmatrix} \begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \\ \vdots \\ \hat{\beta}_n \end{bmatrix}$$

$$= \begin{bmatrix} \hat{\beta}_1 x_{11} + \hat{\beta}_2 x_{12} + \dots + \hat{\beta}_n x_{1n} \\ \hat{\beta}_1 x_{21} + \hat{\beta}_2 x_{22} + \dots + \hat{\beta}_n x_{2n} \\ \vdots \\ \hat{\beta}_1 x_{J1} + \hat{\beta}_2 x_{J2} + \dots + \hat{\beta}_n x_{Jn} \end{bmatrix} = \begin{bmatrix} \hat{v}_1 \\ \hat{v}_2 \\ \vdots \\ \hat{v}_J \end{bmatrix}$$

In R:

```
X %*% beta
```

Practice Question 1

Suppose we estimate the following utility model describing preferences for cars:

$$\tilde{u}_j = \alpha p_j + \beta_1 x_j^{\text{mpg}} + \beta_2 x_j^{\text{elec}} + \varepsilon_j$$

The estimated model produces the following coefficients:

- α: -0.7
- β_1: 0.1
- β_2: -4.0

a) Use the estimated coefficients to compute the WTP for fuel economy and electric car vehicle type.

b) Use the estimated coefficients to compute market shares for the alternatives in the following market:

alternative	price	mpg	elec
1	15	20	0
2	30	100	1
3	20	40	0

Simulating Market Shares with Uncertainty

We can use the coefficients and hessian from a model to obtain draws that reflect parameter uncertainty

```r
beta <- c(-0.7, 0.1, -4.0)

hessian <- matrix(c(
    -6000,   50,   60,
       50, -700,   50,
       60,   50, -300),
    ncol = 3, byrow = TRUE)
```

```r
covariance <- -1*solve(hessian)
draws <- MASS::mvrnorm(10^4, beta,
covariance)

head(draws)
```

```
#>            [,1]       [,2]      [,3]
#> [1,] -0.6905961 0.08251953 -4.026396
#> [2,] -0.6868900 0.13294040 -4.030885
#> [3,] -0.6879329 0.09313779 -3.932620
#> [4,] -0.7046432 0.14688365 -4.102864
#> [5,] -0.6783642 0.11194788 -3.993568
#> [6,] -0.7080309 0.07974313 -3.952816
```

Simulating Market Shares with Uncertainty

Rely on the `predict()` function to compute shares with uncertainty.

Internally, it:

1. Takes draws of $\boldsymbol{\beta}$
2. Computes P_j for each draw
3. Returns mean and confidence interval computed from draws

Simulating Market Shares with Uncertainty

1. Open `logitr-cars`
2. Open `code/7.1-compute-market-sims.R`

Your Turn

20:00

As a team:

1. Develop one or two scenarios pitting your product against one or more competitors.
2. Use one of your estimated models and the `predict()` function to predict market shares for those scenarios.
3. Try plotting your results (see `7.2-plot-market-sims.R` for examples)