EMSE 6035: Marketing of Technology

Modeling Heterogeneous Preferences

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Background: Homogeneous Utility Model

$$\tilde{u}_j = \boldsymbol{\beta}'\mathbf{x}_j + \tilde{\varepsilon}_j$$

$$= \boxed{\beta_1} x_{j1} + \boxed{\beta_2} x_{j2} + \ldots + \tilde{\varepsilon}_j$$

Weights that denote the *relative* value of attributes $x_{j1}, x_{j2}, \ldots$

This model is a *homogeneous* model: everyone has the <u>same</u> coefficients

Est.	Std. Err.
$\hat{\beta}_1$	σ_1
$\hat{\beta}_2$	σ_2
$\vdots$	$\vdots$
$\hat{\beta}_m$	σ_m

Heterogeneous Models

Heterogeneous models allow for *different* people to have *different* coefficients

We will cover two approaches (there are others):

Interaction Models

Group 1		Group 2	
Estimate	Std. Err.	Estimate	Std. Err.
$\hat{\beta}_1$	σ_1	$\hat{\beta}_1$	σ_1
$\hat{\beta}_2$	σ_2	$\hat{\beta}_2$	σ_2
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\hat{\beta}_m$	σ_m	$\hat{\beta}_m$	σ_m

Estimate a “mixed” logit (a.k.a. hierarchical) model

Estimate
$\hat{\beta}_1 \sim \mathrm{N}(\hat{\mu}_1, \hat{\sigma}_1)$
$\hat{\beta}_2 \sim \mathrm{N}(\hat{\mu}_2, \hat{\sigma}_2)$
$\vdots$
$\hat{\beta}_m \sim \mathrm{N}(\hat{\mu}_m, \hat{\sigma}_m)$

Interaction models

Homogeneous model:

$$\tilde{u}_j = \beta_1 x_{j1} + \tilde{\varepsilon}_j$$

Two groups: A & B

$$\tilde{u}_j = \beta_1 x_{j1} + \underline{\beta_2 x_{j1} \delta^{\mathrm{B}}} + \tilde{\varepsilon}_j$$

Where

$\delta^{\mathrm{B}} = 1$ if the person is in group B

$\delta^{\mathrm{B}} = 0$ if the person is in group A

Interaction models

$$\tilde{u}_j = \beta_1 x_{j1} + \beta_2 x_{j1}\delta^{\mathrm{B}} + \tilde{\varepsilon}_j$$
$$= (\beta_1 + \beta_2\delta^{\mathrm{B}})x_{j1} + \tilde{\varepsilon}_j$$

Par.	Interpretation
$\hat{\beta}_1$	Effect of x_{j1} for group A
$\hat{\beta}_2$	*Difference* in effect of x_{j1} between groups A and B

Effect of x_{j1}	
Group A	Group B
$\hat{\beta}_1$	$\hat{\beta}_1 + \hat{\beta}_2$

The scale parameter

$$\tilde{u}_j = \beta_1 x_{j1} + \beta_2 x_{j1} \delta^{\mathrm{B}} + \tilde{\varepsilon}_j$$

$$\tilde{\varepsilon}_j \sim \mathrm{Gumbel}\left(0, \sigma^2 \frac{\pi^2}{6}\right)$$

Scale parameter

$$\frac{\tilde{u}_j}{\cancel{\sigma}} = \frac{1}{\cancel{\sigma}}\left(\beta_1 x_{j1} + \beta_2 x_{j1} \delta^{\mathrm{B}} + \tilde{\varepsilon}_j\right)$$

Assume $\sigma = 1$

$$\tilde{\varepsilon}_j \sim \mathrm{Gumbel}\left(0, \frac{\pi^2}{6}\right)$$

What if we split the data and separately estimate two models:

$$\tilde{u}_j^{\mathrm{A}} = \beta_1 x_{j1} + \tilde{\varepsilon}_j^{\mathrm{A}}$$

$$\tilde{u}_j^{\mathrm{B}} = \beta_1 x_{j1} + \tilde{\varepsilon}_j^{\mathrm{B}}$$

$$\frac{\tilde{u}_j^{\mathrm{A}}}{\sigma^{\mathrm{A}}} = \frac{1}{\sigma^{\mathrm{A}}}\left(\beta_1 x_{j1} + \tilde{\varepsilon}_j^{\mathrm{A}}\right)$$

$$\frac{\tilde{u}_j^{\mathrm{B}}}{\sigma^{\mathrm{B}}} = \frac{1}{\sigma^{\mathrm{B}}}\left(\beta_1 x_{j1} + \tilde{\varepsilon}_j^{\mathrm{B}}\right)$$

Practice Question 1

Suppose we use the following utility model to describe preferences for cars:

$$\tilde{u}_j = \beta_1 x_j^{\text{price}} + \beta_2 x_j^{\text{mpg}} + \beta_3 \delta_j^{\text{elec}} + \varepsilon_j$$

a) Using interactions, write out a model that accounts for differences in the effects of x_j^{price}, x_j^{mpg}, and δ_j^{elec} between two groups: A and B

b) Write out the effects of x_j^{price}, x_j^{mpg}, and δ_j^{elec} for each group.

Use samples of $\widehat{\boldsymbol{\beta}}$ to include uncertainty

Effect of x_{j1}	
Group A	Group B
$\hat{\beta}_1$	$\hat{\beta}_1 + \hat{\beta}_2$

$$\boldsymbol{\beta} \sim \mathrm{N}\left(\widehat{\boldsymbol{\beta}}, \Sigma\right)$$

$$\begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \\ \vdots \\ \hat{\beta}_n \end{bmatrix} \qquad \underbrace{\begin{bmatrix} \sigma_{11}^2 & \cdots & \sigma_{1n}^2 \\ \vdots & \ddots & \vdots \\ \sigma_{1n}^2 & \cdots & \sigma_{jn}^2 \end{bmatrix}}$$

$$-\left[\underbrace{\nabla_{\boldsymbol{\beta}}^2 \ln(\mathcal{L})}_{\text{Hessian}}\right]^{-1}$$

Example in *R*:

```
> library(MASS)
> beta = c(b1 = -0.7, b2 = 0.1)
> hessian = matrix(c(
+     -6000,  50,
+      50,   -6000),
+      ncol=2, byrow=T)
> covariance = -1*(solve(hessian))
> draws  = as.data.frame(mvrnorm(10^5, beta,
covariance))
> b1_A = draws$b1
> b1_B = draws$b1 + draws$b2
> mean(b1_A)
[1] -0.7000618
> mean(b1_B)
[1] -0.6000408
> quantile(b1_A, c(0.025, 0.975))
      2.5%      97.5%
-0.7253724 -0.6748339
> quantile(b1_B, c(0.025, 0.975))
      2.5%      97.5%
-0.6358317 -0.5642215
```

Practice Question 2

Suppose we estimate the following utility model describing preferences for chocolate bars between two groups: A & B

$$\tilde{u}_j = \beta_1 x_j^{\text{price}} + \beta_2 x_j^{\text{cacao}} + \beta_3 x_j^{\text{price}} \delta_j^{\text{B}} + \beta_4 x_j^{\text{cacao}} \delta_j^{\text{B}} + \varepsilon_j$$

The estimated model produces the following coefficients:

Parameter	Coef.
β_1	-0.7
β_2	0.1
β_3	0.2
β_4	0.8

Hessian			
-6000	50	60	70
50	-700	50	100
60	50	-300	20
70	100	20	-6000

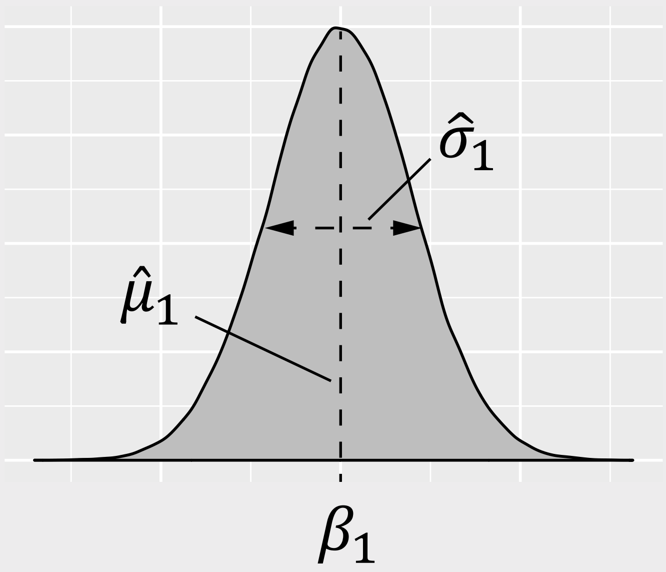
a) Use the `mvrnorm()` function from the `MASS` library to generate 10,000 draws of the model coefficients.
b) Use the draws to compute the mean and 95% confidence intervals of the effects of x_j^{price} and x_j^{cacao} for each group (A & B).

Mixed logit models

Homogeneous model:

$$\tilde{u}_j = \beta_1 x_{j1} + \tilde{\varepsilon}_j$$

In mixed logit, we assume that β_1 is distributed across the population:



Parameter	Estimate
β_1	$\hat{\mu}_1$
	$\hat{\sigma}_1$

The logitr package includes mixed logit functionality

Standard Logit

$$P_j = \frac{e^{v_j}}{\sum_{k=1}^{J} e^{v_k}}$$

Mixed Logit

$$P_j = \int \left(\frac{e^{v_j}}{\sum_{k=1}^{J} e^{v_k}} \right) f(\beta) d\beta$$

To estimate mixed logit models, we use simulation:

1. Draw a value of β from $f(\beta)$, label it β^r
2. Calculate the standard logit fraction using β^r:

$$P_j^r = \frac{e^{v_j^r}}{\sum_{k=1}^{J} e^{v_k^r}}$$

3. Repeat steps 1 & 2 R times and average the results:

$$\widehat{P_j} = \frac{1}{R} \sum_{r=1}^{R} P_j^r$$

Example in *R*:

$$\tilde{u}_j = \beta_1 x_j^{\text{price}} + \beta_2 x_j^{\text{cacao}} + \varepsilon_j$$

$$\beta_1 \sim \mathrm{N}(\mu_1, \sigma_1), \qquad \beta_2 \sim \mathrm{N}(\mu_2, \sigma_2)$$

```
library(logitr)

model = logitr(
    data    = data,
    outcome = 'choice',
    obsID   = 'obsID',
    pars = c('price', 'cacao'),
    randPars = c(price = 'n', cacao = 'n’)
)
```

Practice Question 3

a) Load the `logitr` package, which loads the `yogurt` data frame.

b) Use the `logitr` package to estimate the following homogeneous model:

$$\tilde{u}_j = \beta_1 x_j^{\text{price}} + \beta_2 \delta_j^{\text{feat}} + \beta_3 \delta_j^{\text{dannon}} + \beta_4 \delta_j^{\text{highland}} + \beta_5 \delta_j^{\text{weight}} + \varepsilon_j$$

where the three δ coefficients are dummies for Dannon, Highland, and Weight Watchers brands and Yoplait is the baseline brand.

c) Use the `logitr` package to estimate the same model but with the following mixing distributions:

$$\beta_1 \sim \mathrm{N}(\mu_1, \sigma_1)$$
$$\beta_2 \sim \mathrm{N}(\mu_2, \sigma_2)$$