# Week 9: *Uncertainty*

EMSE 6035: Marketing Analytics for Design Decisions

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October 26, 2022

# Pilot Analysis Report

Assignment is now posted

Due 11/06 (that's 10 days from now)

# Week 9: *Uncertainty*

1. Computing uncertainty

2. Reshaping data

BREAK

3. Cleaning pilot data

4. Estimating pilot data models

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# Maximum likelihood estimation

$$\tilde{u}_j = \boldsymbol{\beta}'\mathbf{x}_j + \tilde{\varepsilon}_j$$

$$= \boxed{\beta_1} x_{j1} + \boxed{\beta_2} x_{j2} + \ldots + \tilde{\varepsilon}_j$$

Weights that denote the *relative* value of attributes $x_{j1}, x_{j2}, \ldots$

Estimate $\beta_1$ , $\beta_2$ , … , by minimizing the negative log-likelihood function:

$$\text{minimize} - \ln(\mathcal{L}) = -\sum_{j=1}^{J} y_j \ln[P_j(\boldsymbol{\beta}|\mathbf{x})]$$

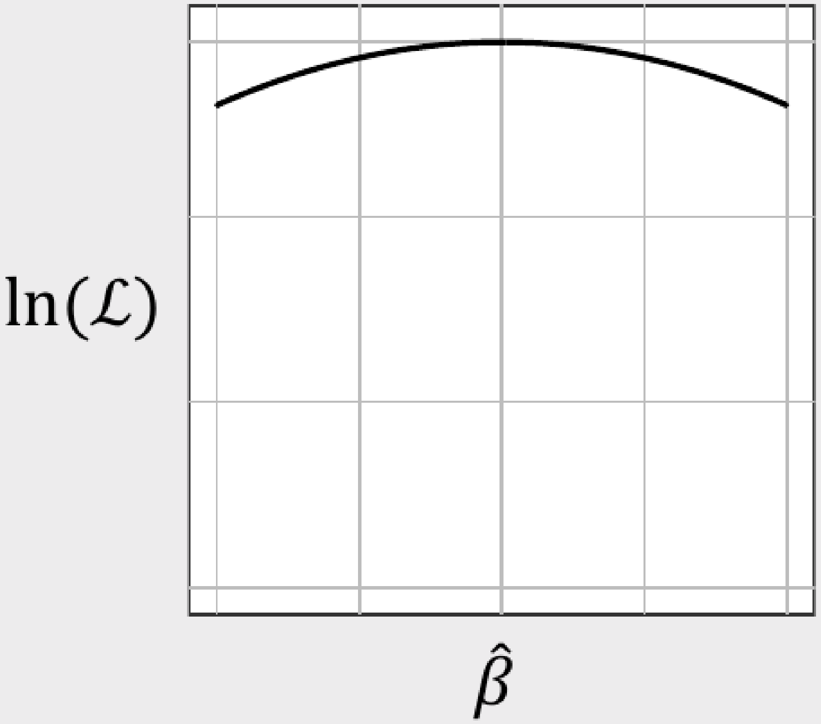
with respect to $\boldsymbol{\beta}$

$y_j = 1$ if alternative $j$ was chosen

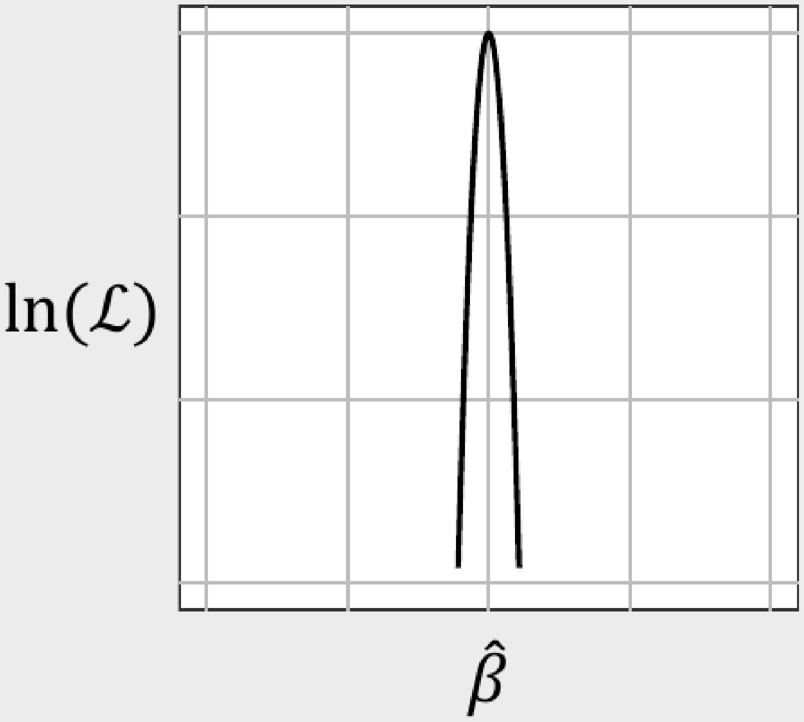
$y_j = 0$ if alternative $j$ was not chosen

# The certainty of $\widehat{\boldsymbol{\beta}}$ is inversely related to the curvature of the log-likelihood function

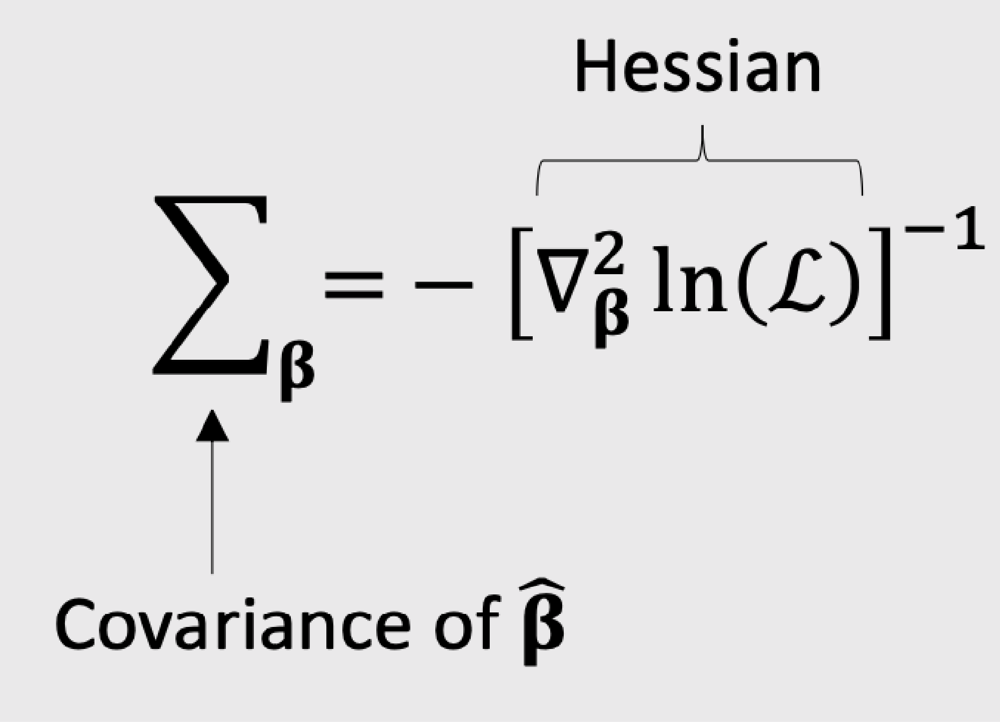
Greater variance in $\ln(\mathcal{L})$,
Less certainty in $\hat{\beta}$



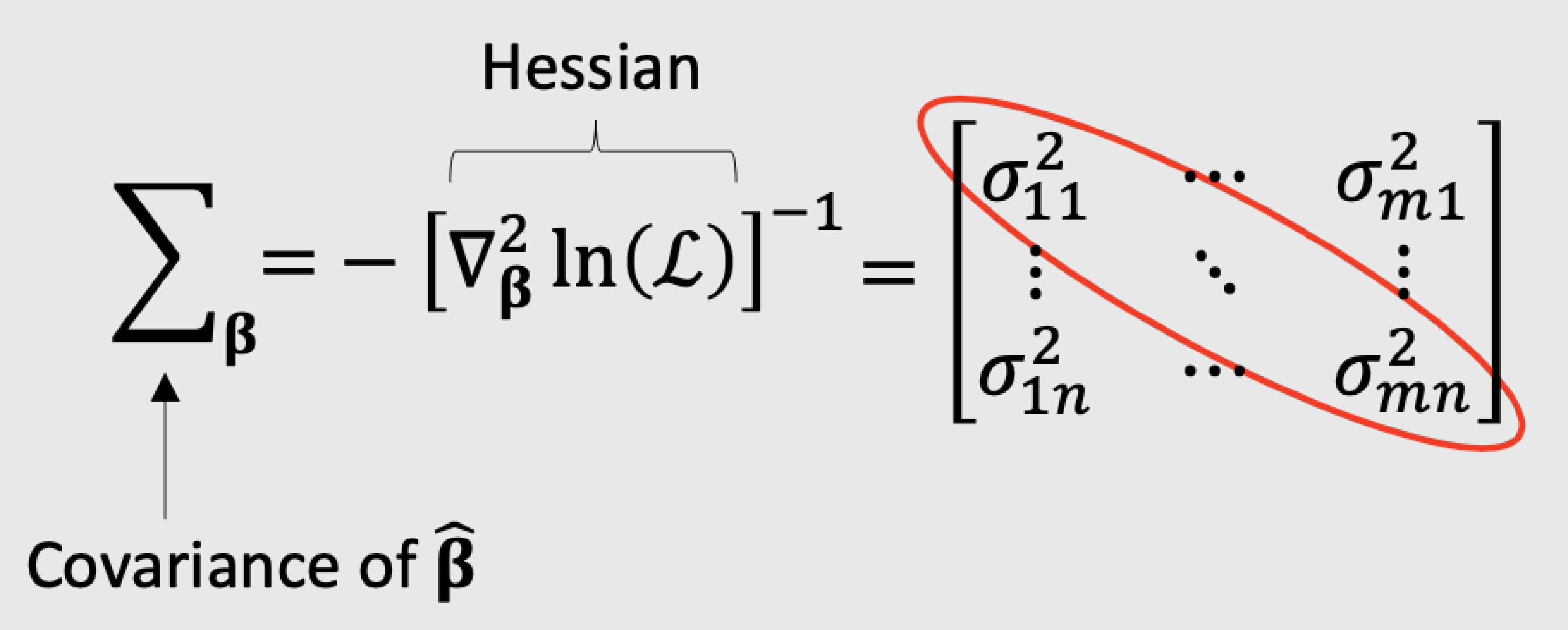

Less variance in $\ln(\mathcal{L})$,
Greater certainty in $\hat{\beta}$



The *curvature* of the log-likelihood function is related to the hessian

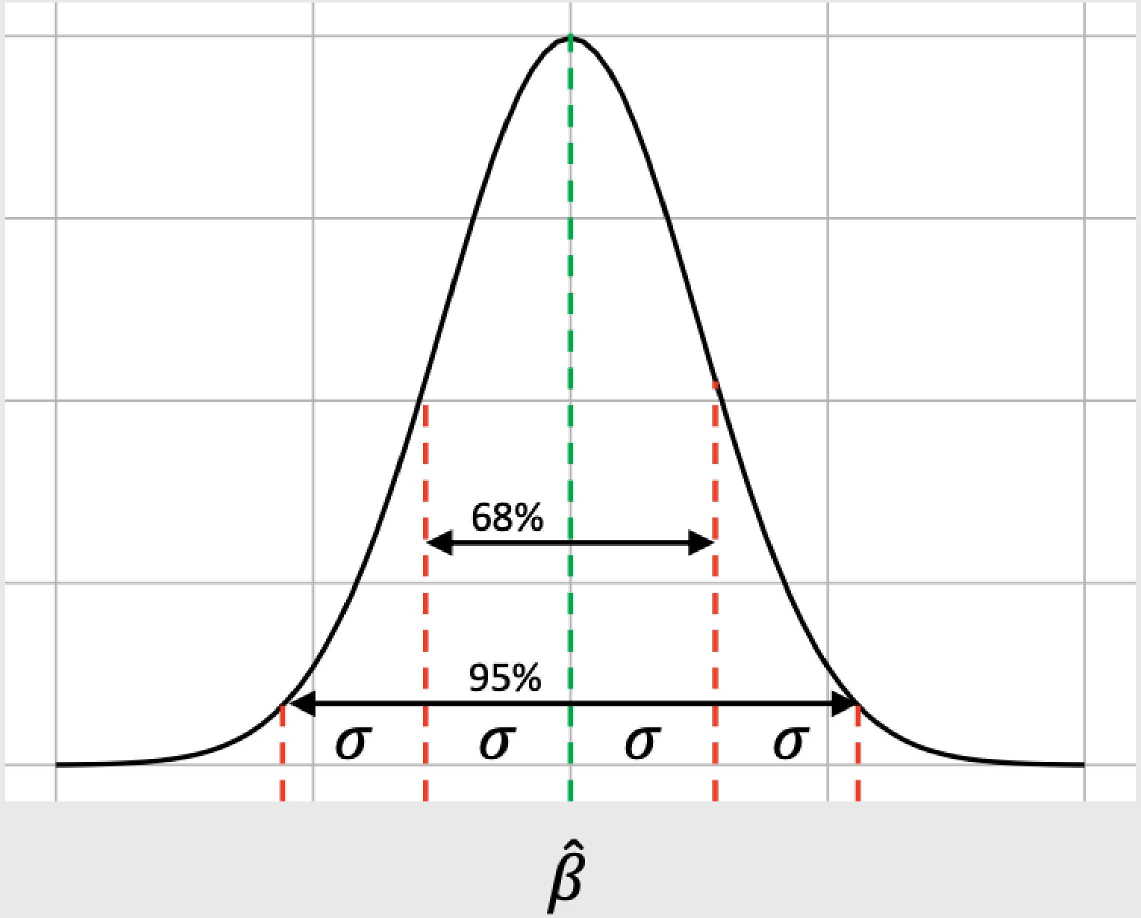


# The *curvature* of the log-likelihood function is related to the hessian



# Usually report parameter uncertainty ("standard errors") with $\sigma$ values

| Est. | Std. Err. |
|---|---|
| $\hat{\beta}_1$ | $\sigma_1$ |
| $\hat{\beta}_2$ | $\sigma_2$ |
| $\vdots$ | $\vdots$ |
| $\hat{\beta}_m$ | $\sigma_m$ |




A 95% confidence interval is approximately $[\hat{\beta} - 2\sigma, \hat{\beta} + 2\sigma]$

# Practice Question 1

Suppose we estimate a model and get the following results:

$$\hat{\beta} = \begin{bmatrix} -0.4 \\ 0.5 \end{bmatrix}$$

$$\nabla^2_{\beta} \ln(\mathcal{L}) = \begin{bmatrix} -6000 & 60 \\ 60 & -700 \end{bmatrix}$$

a) Use the hessian to compute the standard errors for $\hat{\beta}$

b) Use the standard errors to compute a 95% confidence interval around $\hat{\beta}$

# Simulating uncertainty

We can use the coefficients and hessian from a model to obtain draws that reflect parameter uncertainty

```r
beta <- c(-0.7, 0.1, -4.0)

hessian <- matrix(c(
    -6000,   50,   60,
       50, -700,   50,
       60,   50, -300),
    ncol = 3, byrow = TRUE)
```

```r
covariance <- -1*solve(hessian)
draws <- MASS::mvrnorm(10^5, beta,
covariance)

head(draws)
```

```
#>            [,1]       [,2]      [,3]
#> [1,] -0.6871402 0.08833723 -4.090418
#> [2,] -0.7042566 0.08283483 -3.931904
#> [3,] -0.7015105 0.10568167 -4.079077
#> [4,] -0.7063528 0.10164155 -4.047741
#> [5,] -0.6923982 0.07839142 -3.915667
#> [6,] -0.6901872 0.08984027 -3.970755
```

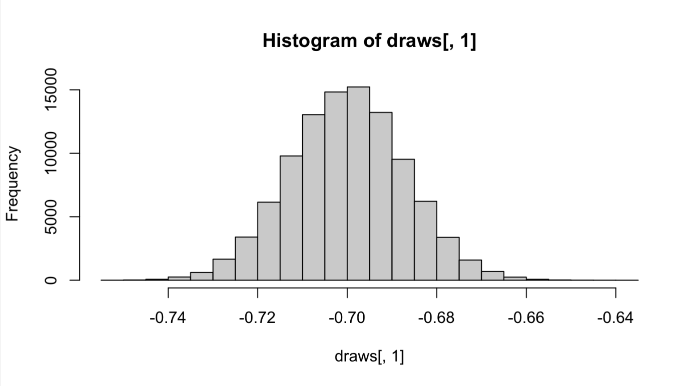
# Simulating uncertainty

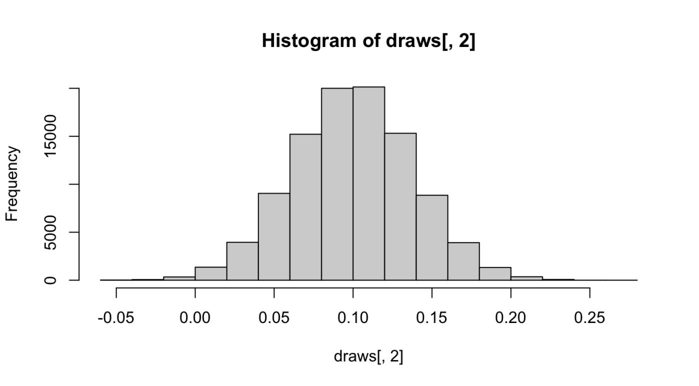
We can use the coefficients and hessian from a model to obtain draws that reflect parameter uncertainty

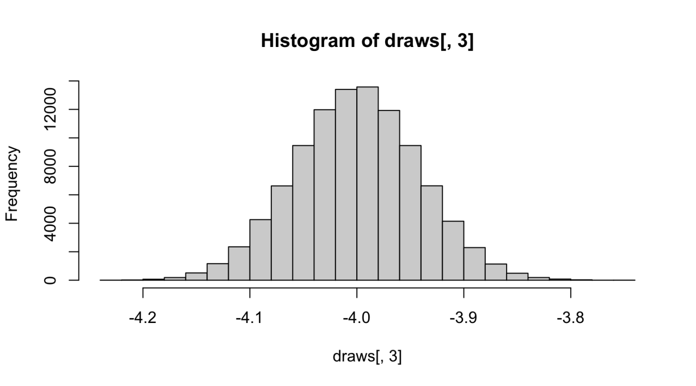
```r
hist(draws[, 1])
```

```r
hist(draws[, 2])
```

```r
hist(draws[, 3])
```



# Practice Question 2

Suppose we estimate the following utility model describing preferences for cars:

$$u_j = \alpha p_j + \beta_1 x_j^{mpg} + \beta_2 x_j^{elec} + \varepsilon_j$$

a) Generate 10,000 draws of the model coefficients using the estimated coefficients and hessian. Use the `mvrnorm()` function from the `MASS` library.

b) Use the draws to compute the mean and 95% confidence intervals of each parameter estimate.

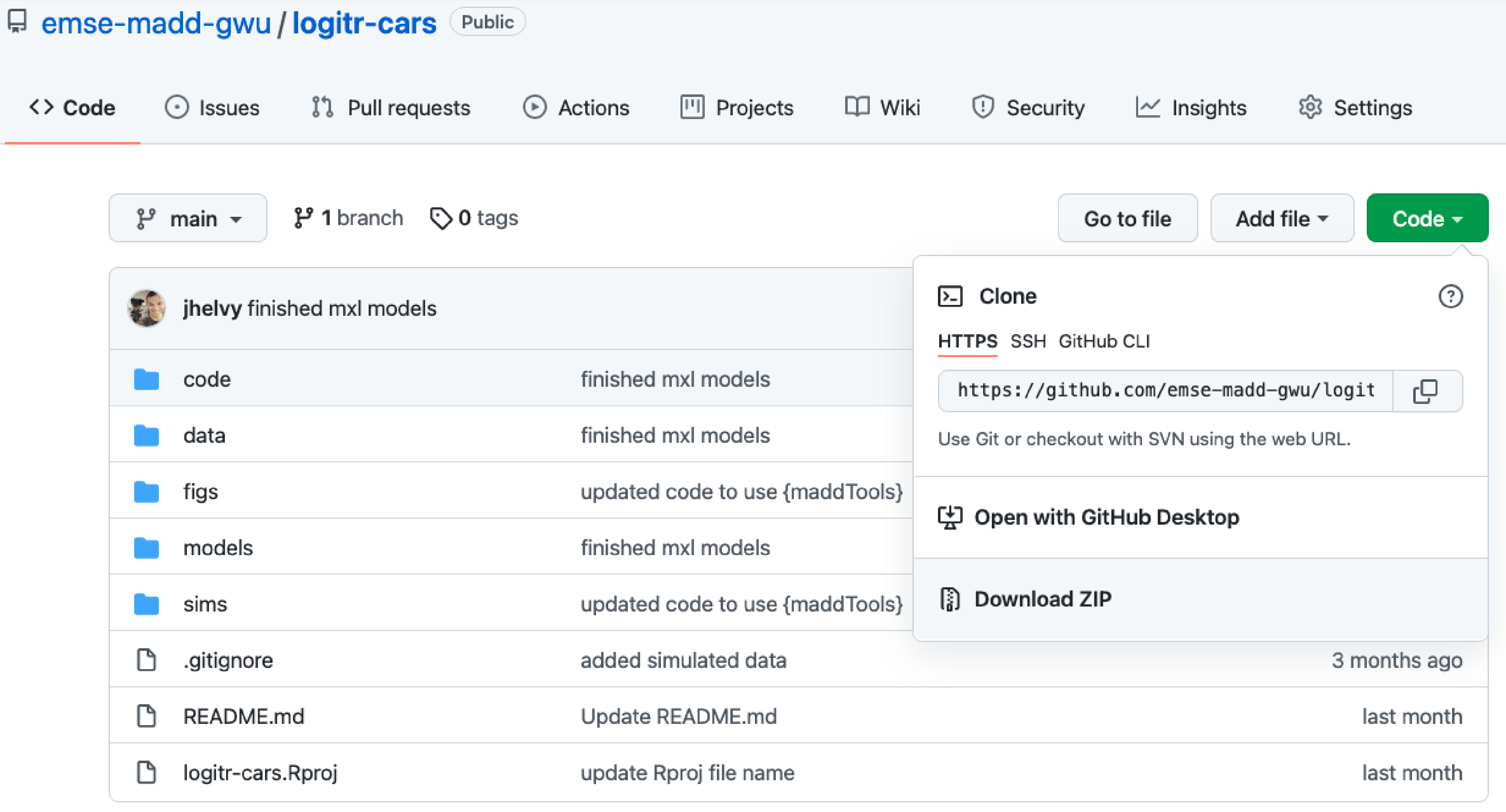
The estimated model produces the following results:

| **Parameter** | **Coefficient** |
|---|---|
| $\alpha$ | -0.7 |
| $\beta_1$ | 0.1 |
| $\beta_2$ | -0.4 |

Hessian:

$$\begin{bmatrix} -6000 & 50 & 60 \\ 50 & -700 & 50 \\ 60 & 50 & -300 \end{bmatrix}$$

# Download the logitr-cars repo from GitHub



# Computing and visualizing uncertainty

1. Open `logitr-cars`
2. Open `code/5.1-uncertainty.R`

# Week 9: *Uncertainty*

1. Computing uncertainty

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BREAK

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# Federal R&D Spending by Department

```
#> # A tibble: 6 × 15
#>    year   DHS   DOC   DOD   DOE   DOT   EPA   HHS
#>   <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
#> 1  1976     0   819 35696 10882  1142   968  9226
#> 2  1977     0   837 37967 13741  1095   966  9507
#> 3  1978     0   871 37022 15663  1156  1175 10533
#> 4  1979     0   952 37174 15612  1004  1102 10127
#> 5  1980     0   945 37005 15226  1048   903 10045
#> 6  1981     0   829 41737 14798   978   901  9644
#>   Interior  NASA   NIH   NSF Other  USDA    VA
#>      <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
#> 1     1152 12513  8025  2372  1191  1837   404
#> 2     1082 12553  8214  2395  1280  1796   374
#> 3     1125 12516  8802  2446  1237  1962   356
#> 4     1176 13079  9243  2404  2321  2054   353
#> 5     1082 13837  9093  2407  2468  1887   359
#> 6      990 13276  8580  2300  1925  1964   382
```

# Federal R&D Spending by Department

## "Wide" format

```
#> # A tibble: 6 × 15
#>    year   DHS   DOC   DOD   DOE   DOT   EPA   HHS
#>   <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
#> 1  1976     0   819 35696 10882  1142   968  9226
#> 2  1977     0   837 37967 13741  1095   966  9507
#> 3  1978     0   871 37022 15663  1156  1175 10533
#> 4  1979     0   952 37174 15612  1004  1102 10127
#> 5  1980     0   945 37005 15226  1048   903 10045
#> 6  1981     0   829 41737 14798   978   901  9644
#>   Interior  NASA   NIH   NSF Other  USDA    VA
#>      <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
#> 1     1152 12513  8025  2372  1191  1837   404
#> 2     1082 12553  8214  2395  1280  1796   374
#> 3     1125 12516  8802  2446  1237  1962   356
#> 4     1176 13079  9243  2404  2321  2054   353
#> 5     1082 13837  9093  2407  2468  1887   359
#> 6      990 13276  8580  2300  1925  1964   382
```

## "Long" format

```
#> # A tibble: 6 × 3
#>   department  year rd_budget_mil
#>   <chr>      <dbl>         <dbl>
#> 1 DOD         1976         35696
#> 2 NASA        1976         12513
#> 3 DOE         1976         10882
#> 4 HHS         1976          9226
#> 5 NIH         1976          8025
#> 6 NSF         1976          2372
```

# Federal R&D Spending by Department

## "Wide" format

```
#> # A tibble: 6 × 15
#>    year   DHS   DOC   DOD   DOE   DOT   EPA   HHS
#>   <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
#> 1  1976     0   819 35696 10882  1142   968  9226
#> 2  1977     0   837 37967 13741  1095   966  9507
#> 3  1978     0   871 37022 15663  1156  1175 10533
#> 4  1979     0   952 37174 15612  1004  1102 10127
#> 5  1980     0   945 37005 15226  1048   903 10045
#> 6  1981     0   829 41737 14798   978   901  9644
#>   Interior  NASA   NIH   NSF Other  USDA    VA
#>      <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
#> 1     1152 12513  8025  2372  1191  1837   404
#> 2     1082 12553  8214  2395  1280  1796   374
#> 3     1125 12516  8802  2446  1237  1962   356
#> 4     1176 13079  9243  2404  2321  2054   353
#> 5     1082 13837  9093  2407  2468  1887   359
#> 6      990 13276  8580  2300  1925  1964   382
```

## "Long" format

```
#> # A tibble: 6 × 3
#>   department  year rd_budget_mil
#>   <chr>      <dbl>         <dbl>
#> 1 DOD         1976         35696
#> 2 NASA        1976         12513
#> 3 DOE         1976         10882
#> 4 HHS         1976          9226
#> 5 NIH         1976          8025
#> 6 NSF         1976          2372
```

```
#> [1] 588   3
```

# Tidy data = "Long" format

- Each **variable** has its own **column**
- Each **observation** has its own **row**

# Tidy data

- Each **variable** has its own **column**
- Each **observation** has its own **row**

```
#> # A tibble: 6 × 3
#>   department  year rd_budget_mil
#>   <chr>      <dbl>         <dbl>
#> 1 DOD         1976         35696
#> 2 NASA        1976         12513
#> 3 DOE         1976         10882
#> 4 HHS         1976          9226
#> 5 NIH         1976          8025
#> 6 NSF         1976          2372
```

# "Long" format

```
#> # A tibble: 6 × 3
#>   department  year rd_budget_mil
#>   <chr>      <dbl>         <dbl>
#> 1 DOD         1976         35696
#> 2 NASA        1976         12513
#> 3 DOE         1976         10882
#> 4 HHS         1976          9226
#> 5 NIH         1976          8025
#> 6 NSF         1976          2372
```

# "Wide" format

```
#> # A tibble: 6 × 15
#>    year   DHS   DOC   DOD   DOE   DOT   EPA   HHS
#>   <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
#> 1  1976     0   819 35696 10882  1142   968  9226
#> 2  1977     0   837 37967 13741  1095   966  9507
#> 3  1978     0   871 37022 15663  1156  1175 10533
#> 4  1979     0   952 37174 15612  1004  1102 10127
#> 5  1980     0   945 37005 15226  1048   903 10045
#> 6  1981     0   829 41737 14798   978   901  9644
#>   Interior  NASA   NIH   NSF Other  USDA    VA
#>      <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
#> 1     1152 12513  8025  2372  1191  1837   404
#> 2     1082 12553  8214  2395  1280  1796   374
#> 3     1125 12516  8802  2446  1237  1962   356
#> 4     1176 13079  9243  2404  2321  2054   353
#> 5     1082 13837  9093  2407  2468  1887   359
#> 6      990 13276  8580  2300  1925  1964   382
```

# Do the names describe the values?

**Yes**: "Long" format

```
#> # A tibble: 6 × 3
#>   department  year rd_budget_mil
#>   <chr>      <dbl>         <dbl>
#> 1 DOD         1976         35696
#> 2 NASA        1976         12513
#> 3 DOE         1976         10882
#> 4 HHS         1976          9226
#> 5 NIH         1976          8025
#> 6 NSF         1976          2372
```

**No**: "Wide" format

```
#> # A tibble: 6 × 8
#>    year   DHS   DOC   DOD   DOE   DOT   EPA   HHS
#>   <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
#> 1  1976     0   819 35696 10882  1142   968  9226
#> 2  1977     0   837 37967 13741  1095   966  9507
#> 3  1978     0   871 37022 15663  1156  1175 10533
#> 4  1979     0   952 37174 15612  1004  1102 10127
#> 5  1980     0   945 37005 15226  1048   903 10045
#> 6  1981     0   829 41737 14798   978   901  9644
```

# **Quick practice 1**: "long" or "wide" format?

**Description**: Tuberculosis cases in various countries

```
#> # A tibble: 6 × 4
#>   country      year  cases population
#>   <chr>       <dbl>  <dbl>      <dbl>
#> 1 Afghanistan  1999    745   19987071
#> 2 Afghanistan  2000   2666   20595360
#> 3 Brazil       1999  37737  172006362
#> 4 Brazil       2000  80488  174504898
#> 5 China        1999 212258 1272915272
#> 6 China        2000 213766 1280428583
```

# **Quick practice 2**: "long" or "wide" format?

**Description**: Word counts in LOTR trilogy

```
#> # A tibble: 9 × 4
#>   Film                        Race   Female  Male
#>   <chr>                       <chr>   <dbl> <dbl>
#> 1 The Fellowship Of The Ring Elf      1229   971
#> 2 The Fellowship Of The Ring Hobbit     14  3644
#> 3 The Fellowship Of The Ring Man         0  1995
#> 4 The Return Of The King     Elf       183   510
#> 5 The Return Of The King     Hobbit      2  2673
#> 6 The Return Of The King     Man       268  2459
#> 7 The Two Towers             Elf       331   513
#> 8 The Two Towers             Hobbit      0  2463
#> 9 The Two Towers             Man       401  3589
```

# **Quick practice 3**: "long" or "wide" format?

**Description**: Word counts in LOTR trilogy

```
#> # A tibble: 15 × 4
#>    Film                       Race   Gender
#>    <chr>                      <chr>  <chr>
#>  1 The Fellowship Of The Ring Elf    Female
#>  2 The Fellowship Of The Ring Elf    Male
#>  3 The Fellowship Of The Ring Hobbit Female
#>  4 The Fellowship Of The Ring Hobbit Male
#>  5 The Fellowship Of The Ring Man    Female
#>  6 The Fellowship Of The Ring Man    Male
#>  7 The Return Of The King     Elf    Female
#>  8 The Return Of The King     Elf    Male
#>  9 The Return Of The King     Hobbit Female
#> 10 The Return Of The King     Hobbit Male
#> 11 The Return Of The King     Man    Female
#> 12 The Return Of The King     Man    Male
#> 13 The Two Towers             Elf    Female
#> 14 The Two Towers             Elf    Male
#> 15 The Two Towers             Hobbit Female
```

# Reshaping data with `pivot_longer()` and `pivot_wider()`

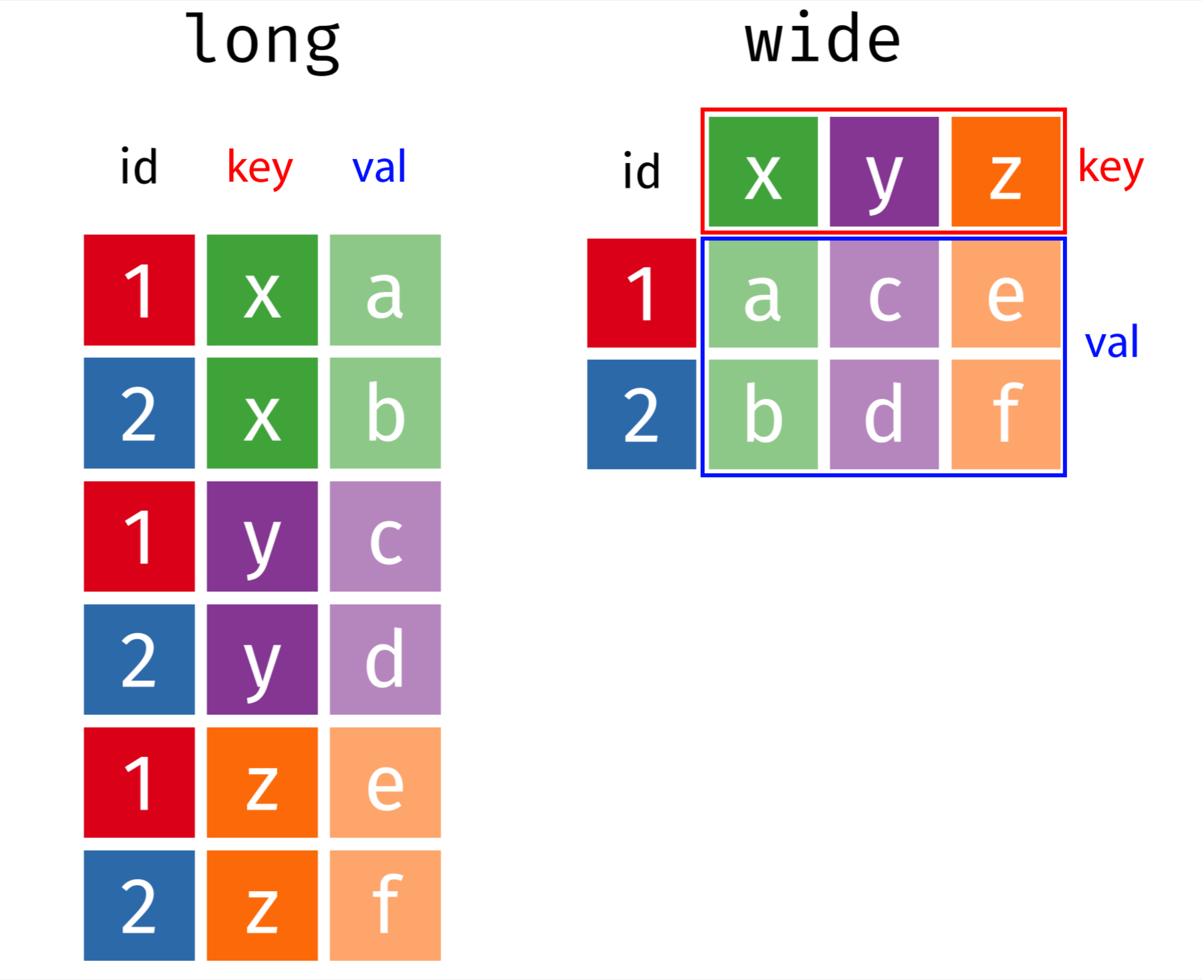
# Reshaping data

`pivot_longer()`
`pivot_wider()`

wide

| id | x | y | z |
|---|---|---|---|
| 1 | a | c | e |
| 2 | b | d | f |

# From "long" to "wide" with `pivot_wider()`



# From "long" to "wide" with `pivot_wider()`

```
head(fed_spend_long)
```

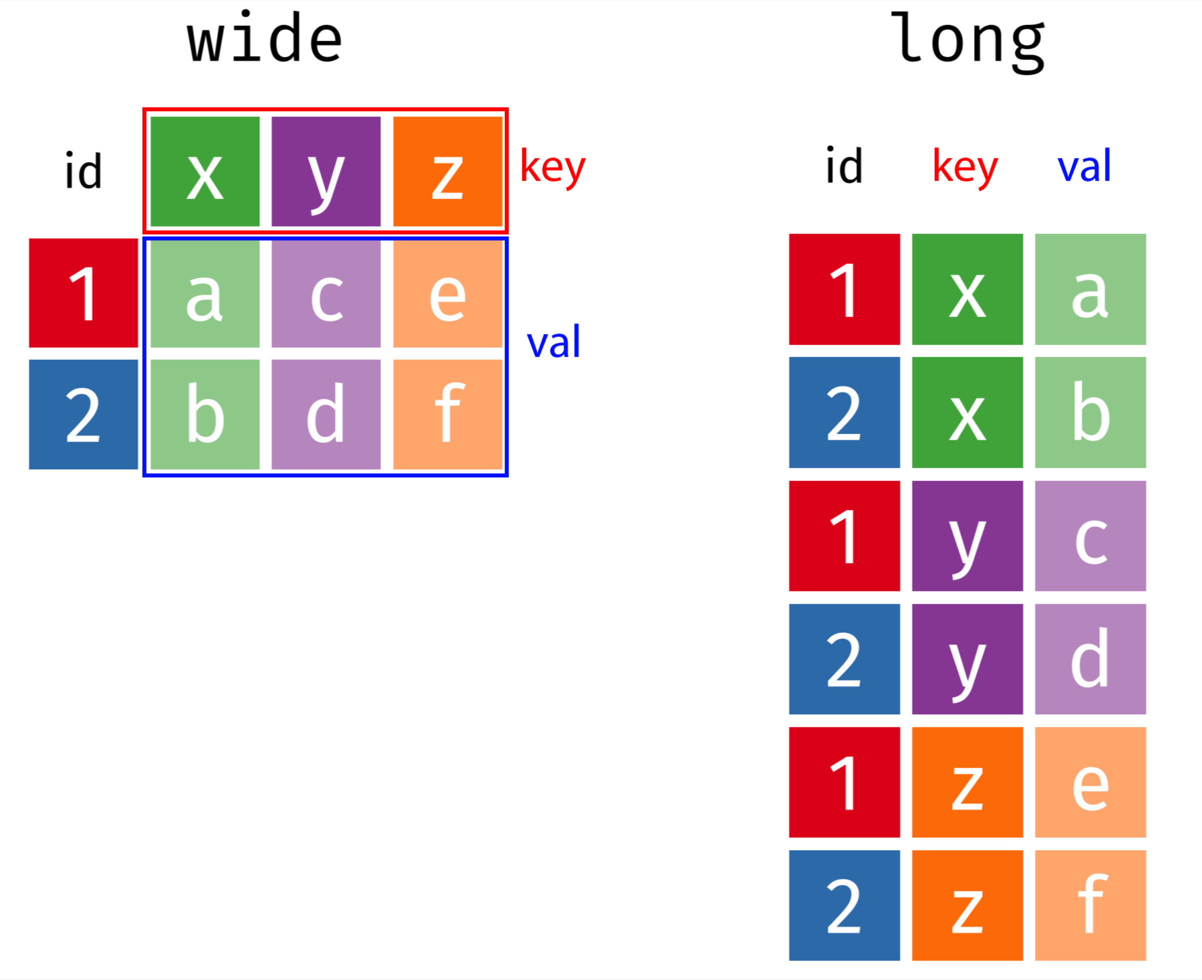
```
#> # A tibble: 6 × 3
#>   department  year rd_budget_mil
#>   <chr>      <dbl>         <dbl>
#> 1 DOD         1976         35696
#> 2 NASA        1976         12513
#> 3 DOE         1976         10882
#> 4 HHS         1976          9226
#> 5 NIH         1976          8025
#> 6 NSF         1976          2372
```

```
fed_spend_wide <- fed_spend_long %>%
    pivot_wider(
        names_from = department,
        values_from = rd_budget_mil)

head(fed_spend_wide)
```

```
#> # A tibble: 6 × 15
#>     year   DOD  NASA   DOE   HHS   NIH   NSF
USDA
#>    <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
<dbl>
#> 1  1976 35696 12513 10882  9226  8025  2372
1837
#> 2  1977 37967 12553 13741  9507  8214  2395
1796
#> 3  1978 37022 12516 15663 10533  8802  2446
1962
#> 4  1979 37174 13079 15612 10127  9243  2404
2054
```

# From "wide" to "long" with `pivot_longer()`



# From "wide" to "long" with `pivot_longer()`

```
head(fed_spend_wide)
```

```
#> # A tibble: 6 × 15
#>     year   DOD  NASA   DOE   HHS
NIH   NSF  USDA
#>    <dbl> <dbl> <dbl> <dbl> <dbl>
<dbl> <dbl> <dbl>
#> 1  1976 35696 12513 10882  9226
8025  2372  1837
#> 2  1977 37967 12553 13741  9507
8214  2395  1796
#> 3  1978 37022 12516 15663 10533
8802  2446  1962
#> 4  1979 37174 13079 15612 10127
9243  2404  2054
#> 5  1980 37005 13837 15226 10045
9093  2407  1887
#> 6  1981 41737 13276 14798  9644
8580  2300  1964
```

```
fed_spend_long <- fed_spend_wide %>%
    pivot_longer(
        names_to = "department",
        values_to = "rd_budget_mil",
        cols = DOD:Other)

head(fed_spend_long)
```

```
#> # A tibble: 6 × 3
#>     year department rd_budget_mil
#>    <dbl> <chr>              <dbl>
#> 1  1976 DOD                35696
#> 2  1976 NASA               12513
#> 3  1976 DOE                10882
#> 4  1976 HHS                 9226
#> 5  1976 NIH                 8025
#> 6  1976 NSF                 2372
```

# Can also set `cols` by selecting which columns *not* to use

```r
names(fed_spend_wide)
```

```
#>  [1] "year"     "DOD"      "NASA"
"DOE"
#>  [5] "HHS"      "NIH"      "NSF"
"USDA"
#>  [9] "Interior" "DOT"      "EPA"
"DOC"
#> [13] "DHS"      "VA"       "Other"
```

```r
fed_spend_long <- fed_spend_wide %>%
    pivot_longer(
        names_to = "department",
        values_to = "rd_budget_mil",
        cols = -year)

head(fed_spend_long)
```

```
#> # A tibble: 6 × 3
#>    year department rd_budget_mil
#>   <dbl> <chr>              <dbl>
#> 1  1976 DOD                35696
#> 2  1976 NASA               12513
#> 3  1976 DOE                10882
#> 4  1976 HHS                 9226
#> 5  1976 NIH                 8025
#> 6  1976 NSF                 2372
```

# Your turn: Reshaping Data

15:00

Open the `practice.Rmd` file.

Run the code chunk to read in the following two data files:

- `pv_cell_production.xlsx`: Data on solar photovoltaic cell production by country
- `milk_production.csv`: Data on milk production by state

Now modify the format of each:

- If the data are in "wide" format, convert it to "long" with `pivot_longer()`
- If the data are in "long" format, convert it to "wide" with `pivot_wider()`

# Break

05:00

# Week 9: *Uncertainty*

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BREAK

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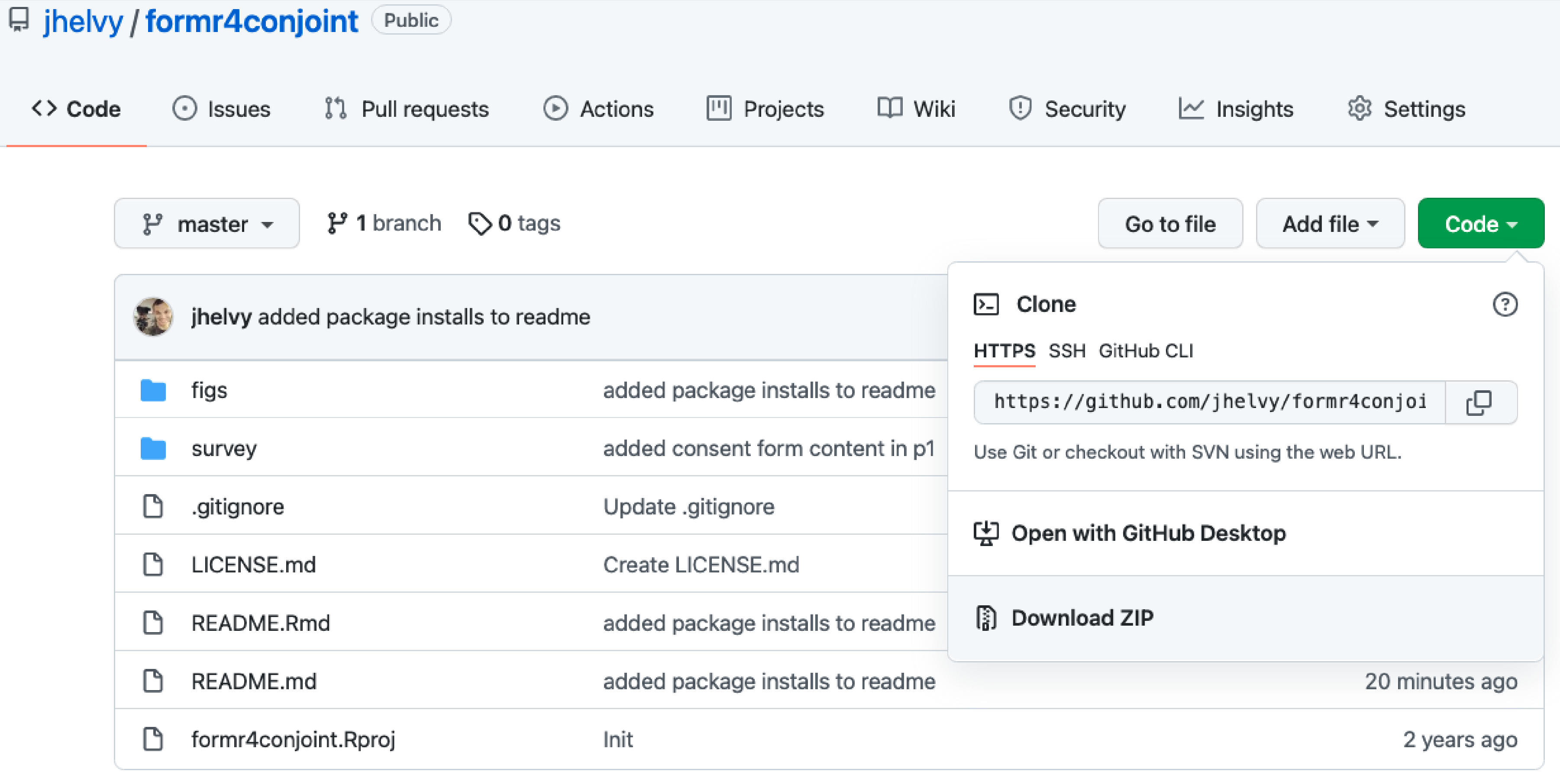
# Download the formr4conjoint repo from GitHub



# Cleaning formr survey data

1. Open `formr4conjoint.Rproj`
2. Open `code/data_cleaning.R`

# Your Turn

`20:00`

As a team, pick up where you left off last week and create a `choiceData` data frame in a "long" format

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# Estimating pilot data models

1. Open `formr4conjoint.Rproj`
2. Open `code/modeling.R`

# Your Turn

## As a team:

1. Use your `choiceData` data frame to estimate preliminary choice models.
2. Interpret your model coefficients with uncertainty.