1. A random variable, $\tilde{x}$, has the probability density function $(\mathrm{PDF}), f_{\tilde{x}}(x)$, shown below.

- What is the approximate probability that $\tilde{x}=0$ ?
- At $x=0$, the PDF is at approximately 0.4 , so there is approximately a $40 \%$ probability that $\tilde{x}=0$.
- What is the approximate probability that $\tilde{x}<0$ ?
- The distribution is symmetric around $\tilde{x}=0$, so the probability that $\tilde{x}<0$ is approximately 0.5 , or a $50 \%$ probability.


2. The chart below shows a profit function, $\pi=f(p)$, where $p$ is price.

- At approximately what price is profit maximized?
- The peak of the curve is at approximately $\$ 5.50$.
- What is the value of $\frac{d f}{d p}(p)$ at that price?
- Since the function is at its maximum value when the price is $\$ 5.50$, the slope at that point should be 0 . Therefore, the first derivative of $f(p), \frac{d f}{d p}(p)$, at the maximizing value of $p$ is 0 .
- Is $\frac{d^{2} f}{d p^{2}}(p)$ positive or negative at that price?
- Since the curve is concave, the second derivative of $f(p), \frac{d^{2} f}{d p^{2}}(p)$, at the maximizing value of $p$ is negative.


3. A student estimates the following linear regression using the motcars dataset in R :

$$
f=\beta_{0}+\beta_{1} w
$$

where $f$ is the vehicle fuel economy (in miles per gallon), and $w$ is the vehicle weight (in $1,000 \mathrm{lbs}$ ). The regression produces the following results:

|  | Estimate | Standard Error | t value | $\operatorname{Pr}(>\|\mathrm{t}\|)$ |
| :---: | :---: | :---: | :---: | :---: |
| $\beta_{0}$ | 37.285 | 1.8776 | 19.858 | $8.24 \mathrm{E}-19$ |
| $\beta_{1}$ | -5.344 | 0.5591 | -9.559 | $1.29 \mathrm{E}-10$ |

- How would you interpret the meaning of the coefficients $\beta_{0}$ and $\beta_{1}$ ?
- The meaning of $\beta_{0}$ is the fuel economy (in mpg ) of a hypothetical vehicle with zero weight. The meaning of $\beta_{1}$ is the decrease in fuel economy (in mpg ) for every increase in $1,000 \mathrm{lbs}$ of a vehicle's weight.
- Use the results of this regression to predict the fuel economy of a car that weighs 4500 pounds.
- To compute the expected fuel economy of a car weighing 4500 lbs , we simply evaluate $f$ using the estimated coefficient values (note that the weight is input at 4.5 since $w$ is in 1000 lbs): $f=37.285-5.344(4.5)=13.237 \mathrm{mpg}$

