Week 12: *Heterogeneity*

EMSE 6035: Marketing Analytics for Design Decisions

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Houskeeping items

- **Final presentations** will be on 12/14 during normal class hours. You can pre-record a video presentation if you want, or you can do it live.
- **Final Analysis** (due 12/10) will also be an html page report.
- I am planning on posting all final analyses (without grades) to the course site as a showcase for future students - **please DM me if you would NOT like your report posted**. (examples from last year)

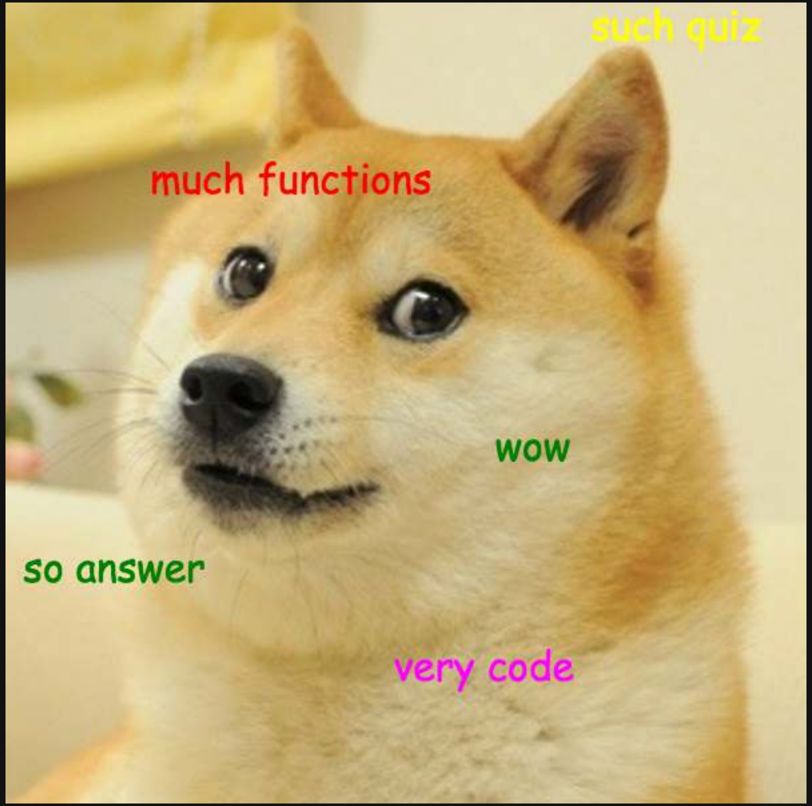
Quiz 5 - last one!

Download the template from the #class channel

Make sure you unzip it!

When done, submit your `quiz5.qmd` on Blackboard

10:00



Week 12: *Heterogeneity*

1. Mixed logit (unobserved heterogeneity)
2. Sub-group modeling (observed heterogeneity)

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Two ways of modeling heterogeneity

"Observed Heterogeneity"

Interaction Models

Group 1		Group 2	
Estimate	Std. Err.	Estimate	Std. Err.
$\hat{\beta}_1$	σ_1	$\hat{\beta}_1$	σ_1
$\hat{\beta}_2$	σ_2	$\hat{\beta}_2$	σ_2
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\hat{\beta}_m$	σ_m	$\hat{\beta}_m$	σ_m

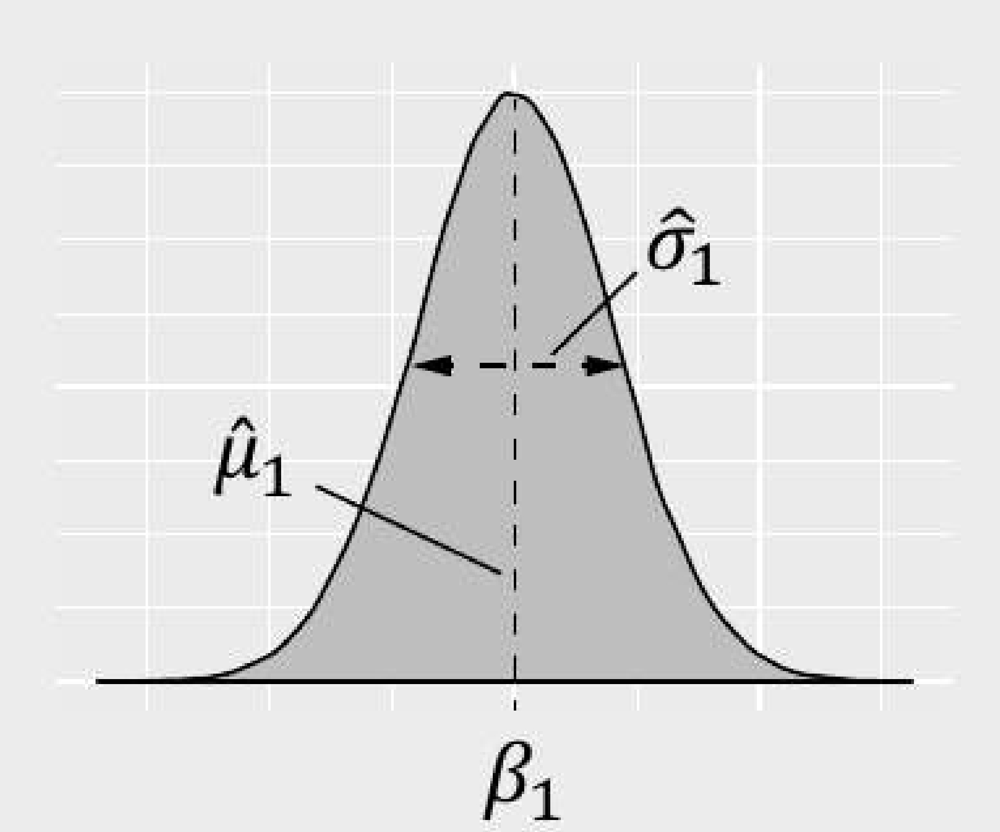
"Unobserved Heterogeneity"

Estimate a "mixed" logit (a.k.a. hierarchical) model

Estimate
$\hat{\beta}_1 \sim \mathrm{N}(\hat{\mu}_1, \hat{\sigma}_1)$
$\hat{\beta}_2 \sim \mathrm{N}(\hat{\mu}_2, \hat{\sigma}_2)$
$\vdots$
$\hat{\beta}_m \sim \mathrm{N}(\hat{\mu}_m, \hat{\sigma}_m)$

Mixed logit

Preference parameters follow a distribution
across sample population



$$\tilde{u}_j = \beta_1 x_j + \varepsilon_j$$

$$\beta_1 \sim \mathrm{N}(\mu_1, \sigma_1)$$

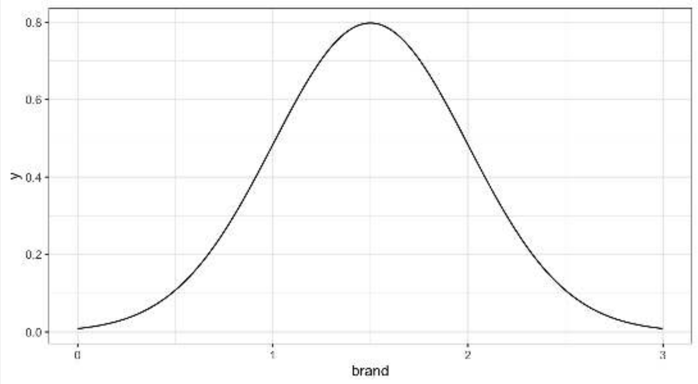
Parameter	Estimate	Standard Error
μ_1	0.1	0.01
σ_1	0.1	0.01

Which distribution should I use?

Normal distribution

When preferences can be positive or negative

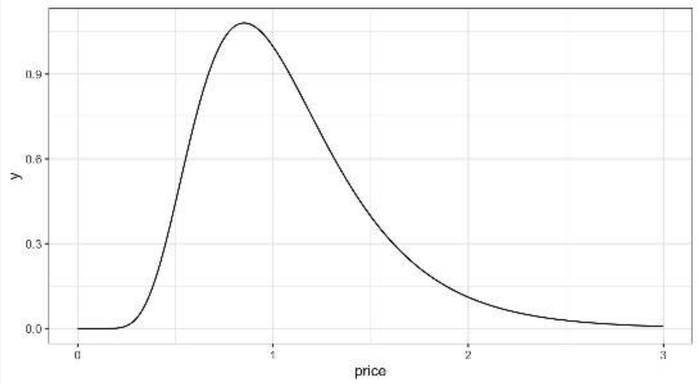
e.g. `brand = "n"`



Log-normal distribution

When preferences should be strictly positive

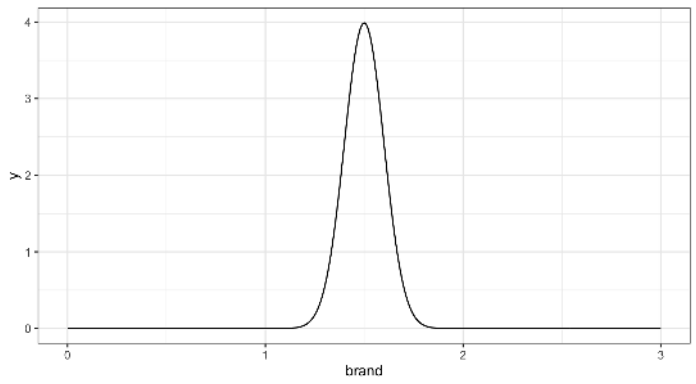
e.g. `-1*price = "ln"`



Fixed parameter

When preferences appear to be homogeneous

(e.g. σ is very small)



Mixed logits are not equivalent in Preference vs. WTP space

Preference space

$$\tilde{u}_j = \alpha p_j + \beta x_j + \varepsilon_j$$

$$\alpha \sim \ln \mathrm{N}(\mu_1, \sigma_1)$$

$$\beta \sim \mathrm{N}(\mu_2, \sigma_2)$$

Mixed logits are not equivalent in Preference vs. WTP space

Preference space

$$\tilde{u}_j = \alpha p_j + \beta x_j + \varepsilon_j$$

$$\alpha \sim \ln \mathrm{N}(\mu_1, \sigma_1)$$

$$\beta \sim \mathrm{N}(\mu_2, \sigma_2)$$

$$\omega = \frac{\beta}{-\alpha} = \frac{\mathrm{N}(\mu_2, \sigma_2)}{-\ln \mathrm{N}(\mu_1, \sigma_1)}$$

WTP space

$$\tilde{u}_j = \lambda(\omega_1 x_j - p_j) + \varepsilon_j$$

$$\omega_1 \sim \mathrm{N}(\mu_1, \sigma_1)$$

Practice Question 3

a) Use the `logitr` package to estimate the following homogeneous model:

$$\tilde{u}_j = \beta_1 x_j^{\text{price}} + \beta_2 \delta_j^{\text{feat}} + \beta_3 \delta_j^{\text{dannon}} + \beta_4 \delta_j^{\text{hiland}} + \beta_5 \delta_j^{\text{weight}} + \varepsilon_j$$

where the three δ coefficients are dummy variables for Dannon, Hiland, and Weight Watchers brands (Yoplait is the reference level).

b) Use the `logitr` package to estimate the same model but with the following mixing distributions:

- $\beta_1 \sim \text{N}(\mu_1, \sigma_1)$
- $\beta_2 \sim \text{N}(\mu_2, \sigma_2)$

Estimating mixed logit models with `logitr`

1. Open `logitr-cars`
2. Open `code/8.1-model-mxl.R`

Your Turn

10:00

As a team, re-estimate the main model you used in your pilot analysis report, but now using a mixed logit model.

Carefully consider which distributions to use (i.e., normal or log-normal) for different variables.

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"Unobserved Heterogeneity"

Estimate a "mixed" logit (a.k.a. hierarchical) model

Estimate
$\hat{\beta}_1 \sim \mathrm{N}(\hat{\mu}_1, \hat{\sigma}_1)$
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$\vdots$
$\hat{\beta}_m \sim \mathrm{N}(\hat{\mu}_m, \hat{\sigma}_m)$

Use interactions to model preferences for multiple groups

Homogenous model:

$$\tilde{u}_j = \beta_1 x_j + \varepsilon_j$$

Two groups: A & B

$$\tilde{u}_j = \beta_1 x_j + \beta_2 x_j \delta^{\mathrm{B}} + \varepsilon_j$$

$$= (\beta_1 + \beta_2 \delta^{\mathrm{B}}) x_j + \varepsilon_j$$

Par.	**Meaning**
β_1	Effect of x_j for group A
β_2	*Difference* in effect of x_j between groups

What's the difference?

Separate models ⊗

$$\tilde{u}_j^{\mathrm{A}} = \beta_1^{\mathrm{A}} x_j + \varepsilon_j^{\mathrm{A}}$$

$$\tilde{u}_j^{\mathrm{B}} = \beta_1^{\mathrm{B}} x_j + \varepsilon_j^{\mathrm{B}}$$

Single model ✓

$$\tilde{u}_j = \beta_1 x_j + \beta_2 x_j \delta^{\mathrm{B}} + \varepsilon_j$$

Accounting for scale differences

Separate models ⊗

$$\tilde{u}_j^{\mathrm{A}} = \alpha^{\mathrm{A}} p_j + \beta_1^{\mathrm{A}} x_j + \varepsilon_j^{\mathrm{A}}$$

$$\tilde{u}_j^{\mathrm{B}} = \alpha^{\mathrm{B}} p_j + \beta_1^{\mathrm{B}} x_j + \varepsilon_j^{\mathrm{B}}$$

Imagine you got the following results

- $\hat{\alpha}^{\mathrm{A}}$ = 100
- $\hat{\beta}^{\mathrm{A}}$ = 200
- $\hat{\alpha}^{\mathrm{B}}$ = 1
- $\hat{\beta}^{\mathrm{B}}$ = 2

Single model ✓

$$\begin{aligned}\tilde{u}_j &= \alpha_1 p_j + \alpha_2 p_j \delta^{\mathrm{B}} + \beta_1 x_j + \beta_2 x_j \delta^{\mathrm{B}} + \varepsilon_j \\ &= (\alpha_1 + \alpha_2 \delta^{\mathrm{B}}) p_j + (\beta_1 + \beta_2 \delta^{\mathrm{B}}) x_j + \varepsilon_j\end{aligned}$$

Accounting for scale differences

Preference Space ⊗

$$\tilde{u}_j^{\mathrm{A}} = \alpha^{\mathrm{A}} p_j + \beta_1^{\mathrm{A}} x_j + \varepsilon_j^{\mathrm{A}}$$

$$\tilde{u}_j^{\mathrm{B}} = \alpha^{\mathrm{B}} p_j + \beta_1^{\mathrm{B}} x_j + \varepsilon_j^{\mathrm{B}}$$

Imagine you got the following results

- $\hat{\alpha}^{\mathrm{A}}$ = 100
- $\hat{\beta}^{\mathrm{A}}$ = 200
- $\hat{\alpha}^{\mathrm{B}}$ = 1
- $\hat{\beta}^{\mathrm{B}}$ = 2

WTP Space ✓

$$\tilde{u}_j^{\mathrm{A}} = \lambda^{\mathrm{A}} (\omega_1^{\mathrm{A}} x_j - p) + \varepsilon_j^{\mathrm{A}}$$

$$\tilde{u}_j^{\mathrm{B}} = \lambda^{\mathrm{B}} (\omega_1^{\mathrm{B}} x_j - p) + \varepsilon_j^{\mathrm{B}}$$

$$\omega = \frac{\beta}{-\alpha}$$

- $\hat{\omega}^{\mathrm{A}} = 200/(-100) = -2$
- $\hat{\omega}^{\mathrm{B}} = 2/(-1) = -2$

Practice Question 1

Suppose we estimate the following utility model describing preferences for cars:

$$\tilde{u}_j = \beta_1 x_j^{\text{price}} + \beta_2 x_j^{\text{mpg}} + \beta_3 x_j^{\text{elec}} + \varepsilon_j$$

a) Using interactions, write out a model that accounts for differences in the effects of x_j^{price}, x_j^{mpg}, and x_j^{elec} between two groups: A and B.

b) Write out the effects of x_j^{price}, x_j^{mpg}, and x_j^{elec} for each group.

Practice Question 2

Suppose we estimate the following utility model describing preferences for chocolate bars between two groups: A & B

$$\tilde{u}_j = \beta_1 x_j^{\text{price}} + \beta_2 x_j^{\text{caco}} + \beta_3 x_j^{\text{price}} \delta_j^{\text{B}} + \beta_4 x_j^{\text{cacao}} \delta_j^{\text{B}} + \varepsilon_j$$

The estimated model produces the following coefficients and hessian:

β = [-0.7, 0.1, 0.2, 0.8]

$$H = \begin{bmatrix} -6000 & 50 & 60 & 70 \\ 50 & -700 & 50 & 100 \\ 60 & 50 & -300 & 20 \\ 70 & 100 & 20 & -6000 \end{bmatrix}$$

a) Use the `mvrnorm()` function from the `MASS` library to generate 10,000 draws of the model coefficients.

b) Use the draws to compute the mean WTP and 95% confidence intervals of the effects of x_j^{price} and x_j^{cacao} for each group (A & B).

Estimating mixed logit models with `logitr`

1. Open `logitr-cars`

2. Open `code/8.2-model-mnl-groups.R`

Your Turn

20:00

Do this individually, and compare with your teammates:

- Examine the demographic and other variables in your pilot data and specify a model that estimates differences between different groups.
- Write code to estimate that model (or multiple models, e.g. WTP space models).
- Compute and compare WTP across the different groups.