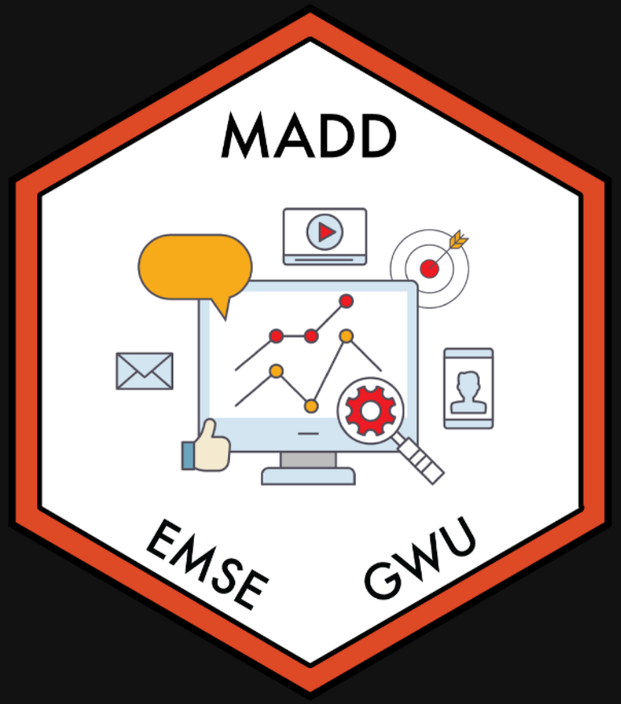


Heterogeneity

EMSE 6035: Marketing Analytics for Design Decisions

John Paul Helveston

November 18, 2026

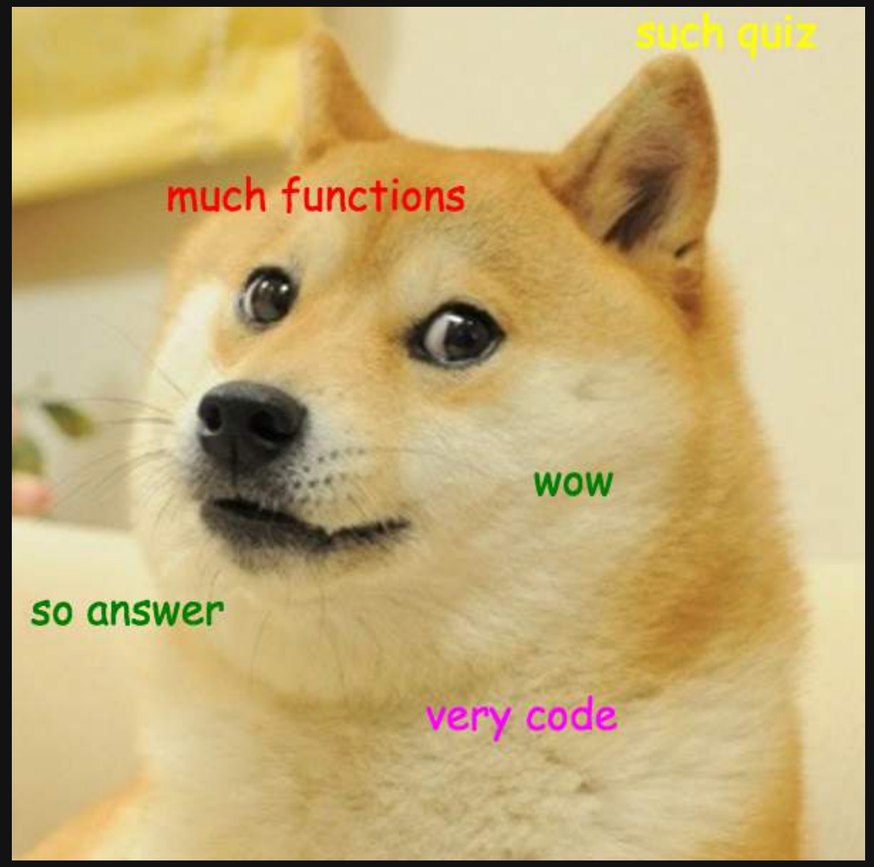
Quiz 5 - last one!

12:00

Write your name on the quiz!

Rules:

- Work alone; no outside help of any kind is allowed.
- No calculators, no notes, no books, no computers, no phones.



Houskeeping items

- **Final presentations** will be on 12/09, 2:30pm - 5:00pm in SEH B1270 (Lehman Auditorium).
- **Final Analysis** (due 12/06) will also be an html page report.
- I am planning on posting all final analyses (without grades) to the course site as a showcase for future students - **please DM me if you would NOT like your report posted**. (examples from last year)

Week 13: *Heterogeneity*

1. Mixed logit (unobserved heterogeneity)

2. Sub-group modeling (observed heterogeneity)

3. Sensitivity analysis

Week 13: *Heterogeneity*

1. Mixed logit (unobserved heterogeneity)
2. Sub-group modeling (observed heterogeneity)
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Two ways of modeling heterogeneity

“Observed Heterogeneity”

“Unobserved Heterogeneity”

Mixed logit

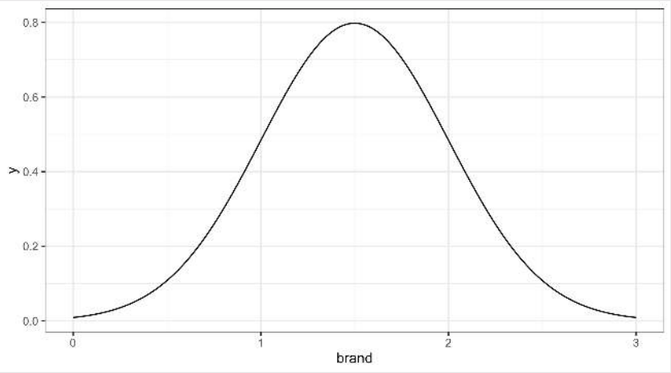
Preference parameters follow a distribution
across sample population

Which distribution should I use?

Normal distribution

When preferences can be positive or negative

e.g. `brand = "n"`



Log-normal distribution

When preferences should be strictly positive

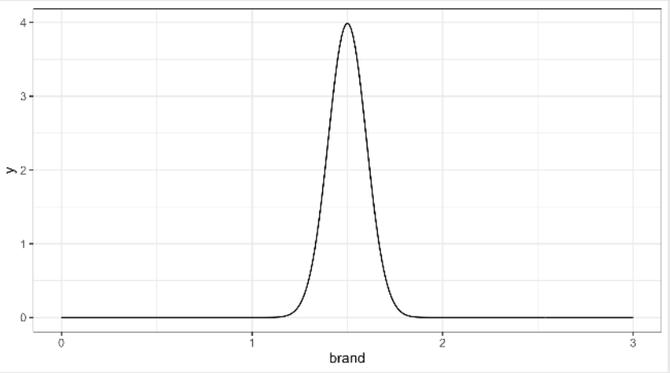
e.g. `-1*price = "ln"`



Fixed parameter

When preferences appear to be homogeneous

(e.g. σ is very small)



Mixed logits are not equivalent in Preference vs. WTP space

Preference space

$$\tilde{u}_j = \alpha p_j + \beta x_j + \varepsilon_j$$

$$\alpha \sim \ln \mathrm{N}(\mu_1, \sigma_1)$$

$$\beta \sim \mathrm{N}(\mu_2, \sigma_2)$$

Mixed logits are not equivalent in Preference vs. WTP space

Preference space

$$\tilde{u}_j = \alpha p_j + \beta x_j + \varepsilon_j$$

$$\alpha \sim \ln \mathrm{N}(\mu_1, \sigma_1)$$

$$\beta \sim \mathrm{N}(\mu_2, \sigma_2)$$

$$\omega = \frac{\beta}{-\alpha} = \frac{\mathrm{N}(\mu_2, \sigma_2)}{-\ln \mathrm{N}(\mu_1, \sigma_1)}$$

WTP space

$$\tilde{u}_j = \lambda(\omega_1 x_j - p_j) + \varepsilon_j$$

$$\omega_1 \sim \mathrm{N}(\mu_1, \sigma_1)$$

Practice Question 3

a. Use the `logitr` package to estimate the following homogeneous model:

$$\tilde{u}_j = \beta_1 x_j^{\text{price}} + \beta_2 \delta_j^{\text{feat}} + \beta_3 \delta_j^{\text{dannon}} + \beta_4 \delta_j^{\text{hiland}} + \beta_5 \delta_j^{\text{weight}} + \varepsilon_j$$

where the three δ coefficients are dummy variables for Dannon, Hiland, and Weight Watchers brands (Yoplait is the reference level).

b. Use the `logitr` package to estimate the same model but with the following mixing distributions:

- $\beta_1 \sim \text{N}(\mu_1, \sigma_1)$
- $\beta_2 \sim \text{N}(\mu_2, \sigma_2)$

Estimating mixed logit models with `logitr`

1. Open `logitr-cars`
2. Open `code/8.1-model-mxl.R`

Your Turn

10:00

As a team, re-estimate the main model you used in your pilot analysis report, but now using a mixed logit model.

Carefully consider which distributions to use (i.e., normal or log-normal) for different variables.

Week 13: *Heterogeneity*

1. Mixed logit (unobserved heterogeneity)
2. Sub-group modeling (observed heterogeneity)
3. Sensitivity analysis

Two ways of modeling heterogeneity

“Observed Heterogeneity”

“Unobserved Heterogeneity”

Use interactions to model preferences for multiple groups

Homogenous model:

$$\tilde{u}_j = \beta_1 x_j + \varepsilon_j$$

Two groups: A & B

$$\begin{aligned}\tilde{u}_j &= \beta_1 x_j + \beta_2 x_j \delta^{\mathrm{B}} + \varepsilon_j \\ &= (\beta_1 + \beta_2 \delta^{\mathrm{B}}) x_j + \varepsilon_j\end{aligned}$$

Par.	Meaning
β_1	Effect of x_j for group A
β_2	*Difference* in effect of x_j between groups

What's the difference?

Separate models ✗

$$\tilde{u}_j^{\mathrm{A}} = \beta_1^{\mathrm{A}} x_j + \varepsilon_j^{\mathrm{A}}$$

$$\tilde{u}_j^{\mathrm{B}} = \beta_1^{\mathrm{B}} x_j + \varepsilon_j^{\mathrm{B}}$$

Single model ✓

$$\tilde{u}_j = \beta_1 x_j + \beta_2 x_j \delta^{\mathrm{B}} + \varepsilon_j$$

Accounting for scale differences

Separate models

$$\tilde{u}_j^{\mathrm{A}} = \alpha^{\mathrm{A}} p_j + \beta_1^{\mathrm{A}} x_j + \varepsilon_j^{\mathrm{A}}$$

$$\tilde{u}_j^{\mathrm{B}} = \alpha^{\mathrm{B}} p_j + \beta_1^{\mathrm{B}} x_j + \varepsilon_j^{\mathrm{B}}$$

Single model

$$\begin{aligned}\tilde{u}_j &= \alpha_1 p_j + \alpha_2 p_j \delta^{\mathrm{B}} + \beta_1 x_j + \beta_2 x_j \delta^{\mathrm{B}} + \varepsilon_j \\ &= (\alpha_1 + \alpha_2 \delta^{\mathrm{B}}) p_j + (\beta_1 + \beta_2 \delta^{\mathrm{B}}) x_j + \varepsilon_j\end{aligned}$$

Imagine you got the following results

- $\hat{\alpha}^{\mathrm{A}}$ = 100
- $\hat{\beta}^{\mathrm{A}}$ = 200
- $\hat{\alpha}^{\mathrm{B}}$ = 1
- $\hat{\beta}^{\mathrm{B}}$ = 2

Accounting for scale differences

Preference Space

$$\tilde{u}_j^{\mathrm{A}} = \alpha^{\mathrm{A}} p_j + \beta_1^{\mathrm{A}} x_j + \varepsilon_j^{\mathrm{A}}$$

$$\tilde{u}_j^{\mathrm{B}} = \alpha^{\mathrm{B}} p_j + \beta_1^{\mathrm{B}} x_j + \varepsilon_j^{\mathrm{B}}$$

Imagine you got the following results

- $\hat{\alpha}^{\mathrm{A}}$ = 100
- $\hat{\beta}^{\mathrm{A}}$ = 200
- $\hat{\alpha}^{\mathrm{B}}$ = 1
- $\hat{\beta}^{\mathrm{B}}$ = 2

WTP Space

$$\tilde{u}_j^{\mathrm{A}} = \lambda^{\mathrm{A}} (\omega_1^{\mathrm{A}} x_j - p) + \varepsilon_j^{\mathrm{A}}$$

$$\tilde{u}_j^{\mathrm{B}} = \lambda^{\mathrm{B}} (\omega_1^{\mathrm{B}} x_j - p) + \varepsilon_j^{\mathrm{B}}$$

$$\omega = \frac{\beta}{-\alpha}$$

- $\hat{\omega}^{\mathrm{A}} = 200/(-100) = -2$
- $\hat{\omega}^{\mathrm{B}} = 2/(-1) = -2$

Practice Question 1

Suppose we estimate the following utility model describing preferences for cars:

$$\tilde{u}_j = \beta_1 x_j^{\text{price}} + \beta_2 x_j^{\text{mpg}} + \beta_3 x_j^{\text{elec}} + \varepsilon_j$$

a. Using interactions, write out a model that accounts for differences in the effects of x_j^{price}, x_j^{mpg}, and x_j^{elec} between two groups: A and B.

b. Write out the effects of x_j^{price}, x_j^{mpg}, and x_j^{elec} for each group.

Practice Question 2

Suppose we estimate the following utility model describing preferences for chocolate bars between two groups: A & B

$$\tilde{u}_j = \beta_1 x_j^{\text{price}} + \beta_2 x_j^{\text{caco}} + \beta_3 x_j^{\text{price}} \delta_j^{\text{B}} + \beta_4 x_j^{\text{cacao}} \delta_j^{\text{B}} + \varepsilon_j$$

The estimated model produces the following coefficients and hessian:

β = [-0.7, 0.1, 0.2, 0.8]

$$H = \begin{bmatrix} -6000 & 50 & 60 & 70 \\ 50 & -700 & 50 & 100 \\ 60 & 50 & -300 & 20 \\ 70 & 100 & 20 & -6000 \end{bmatrix}$$

a. Use the `mvrnorm()` function from the `MASS` library to generate 10,000 draws of the model coefficients.

b. Use the draws to compute the mean WTP and 95% confidence intervals of the effects of x_j^{price} and x_j^{cacao} for each group (A & B).

Estimating mixed logit models with `logitr`

1. Open `logitr-cars`

2. Open `code/8.2-mnl-groups.R`

Your Turn

20:00

Do this individually, and compare with your teammates:

- Examine the demographic and other variables in your pilot data and specify a model that estimates differences between different groups.
- Write code to estimate that model (or multiple models, e.g. WTP space models).
- Compute and compare WTP across the different groups.

Week 13: *Heterogeneity*

1. Mixed logit (unobserved heterogeneity)
2. Sub-group modeling (observed heterogeneity)
3. Sensitivity analysis

What’s a sensitivity analysis?

A market simulation gives you **one** predicted share for **one** market.

The next question your client will ask:

“How much does that share change if we change our product?”

1. Pick a **baseline** market (e.g., your market simulation)
2. Change **one** attribute of your product, holding everything else fixed
3. Re-run `predict()` and see how your product’s share moves

Two common sensitivity analyses

1. Sweep the price

Re-predict your product's share across a range of prices

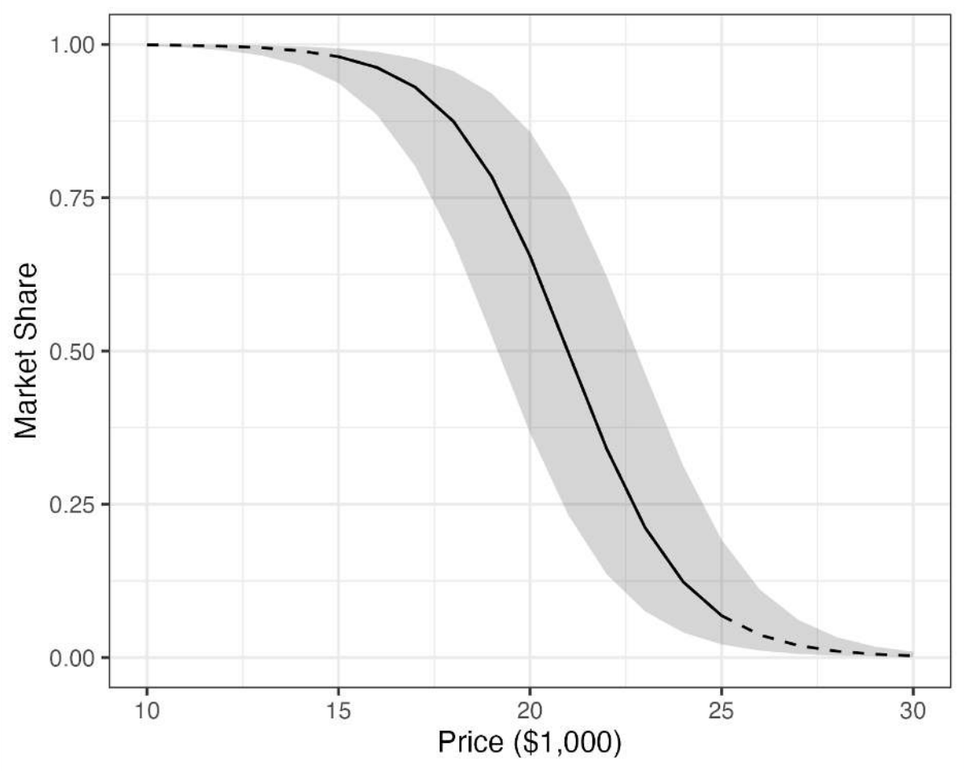
→ **Market share vs. price**
→ **Revenue vs. price** (price x share x market size)

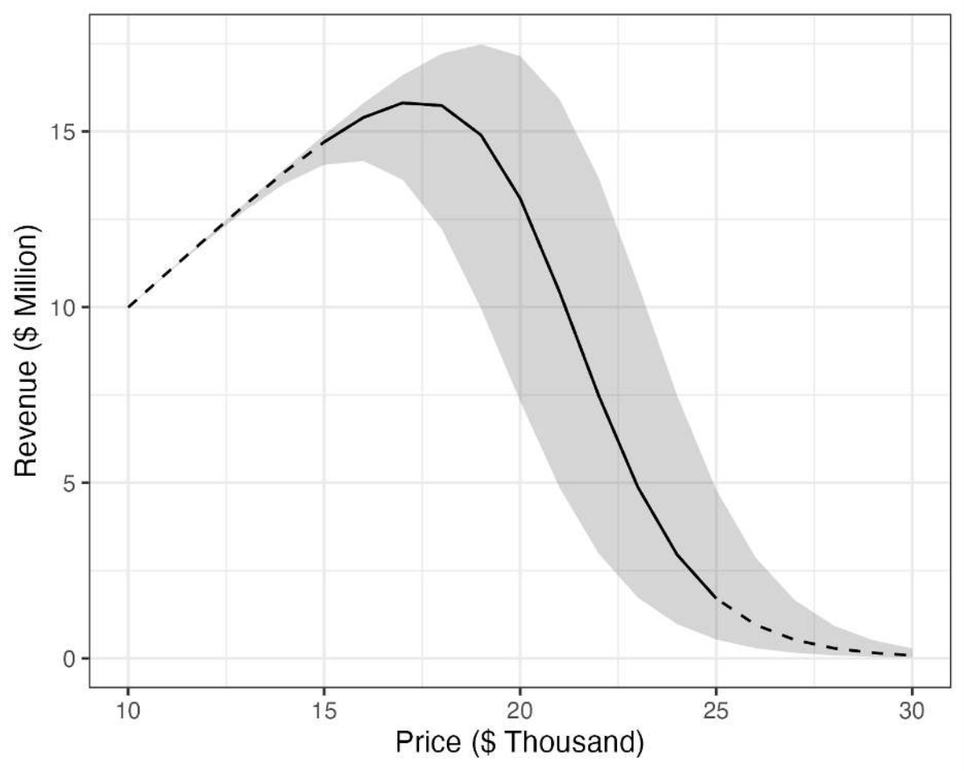
2. Push each attribute up & down

Move each attribute by a fixed amount (e.g. +/- 20%) in the direction that **raises** ("high") and **lowers** ("low") your share

→ **Tornado plot**

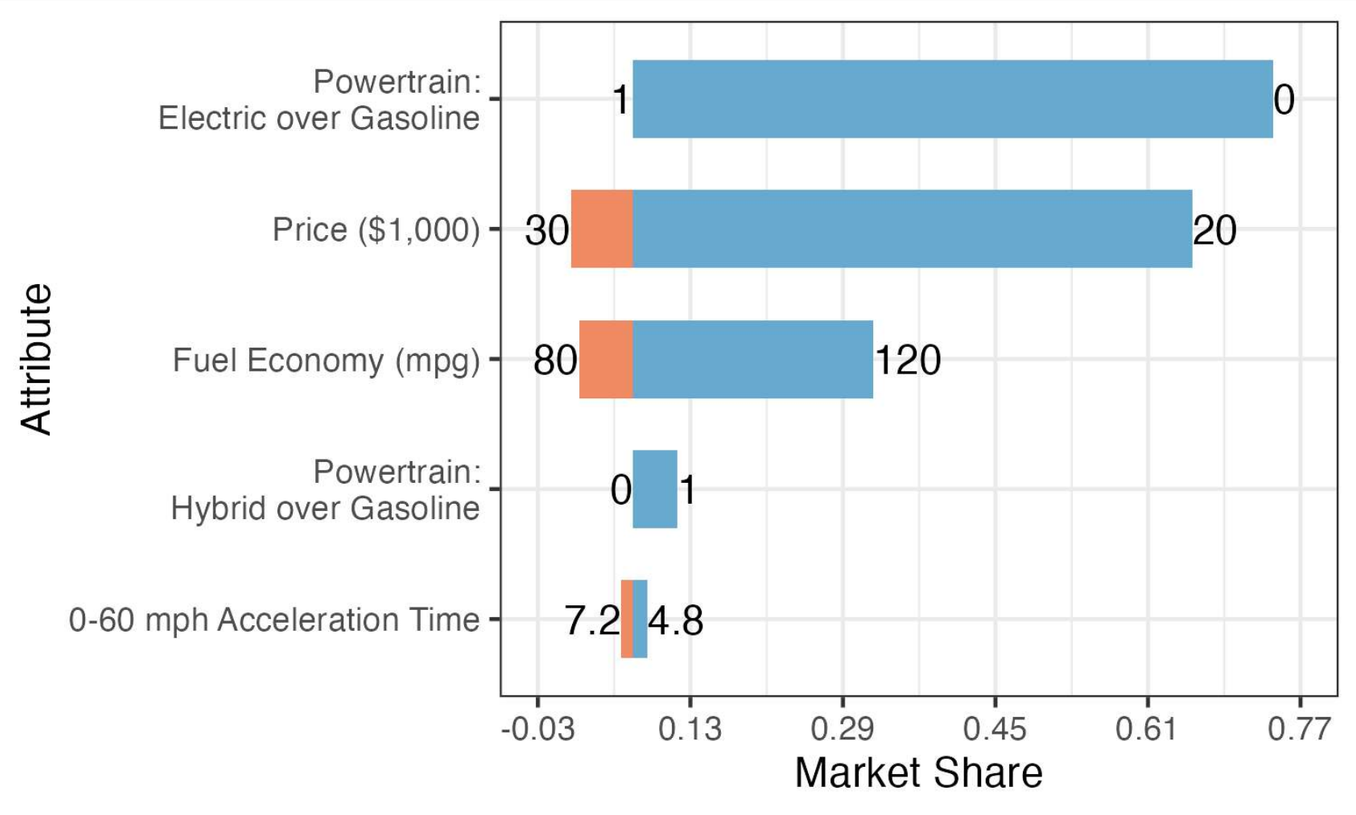
Sensitivity to price





Solid line = prices included in the survey; dashed = extrapolation beyond that range

Sensitivity to other attributes: “tornado” plot



- Each bar is the EV’s share when **one** attribute moves to its low or high value
- Sorted so the attributes your share depends on **most** are on top

Sensitivity analysis with `logitr`

1. Open `logitr-cars`
2. Open `code/9.1-compute-sensitivity.R`
3. Open `code/9.2-plot-sensitivity.R`

These charts belong in your Final Analysis report.
Think about what makes a good **baseline** market for your project.